The Descartes-Cassegrain Telescope

Where a null-test facilitates the things!

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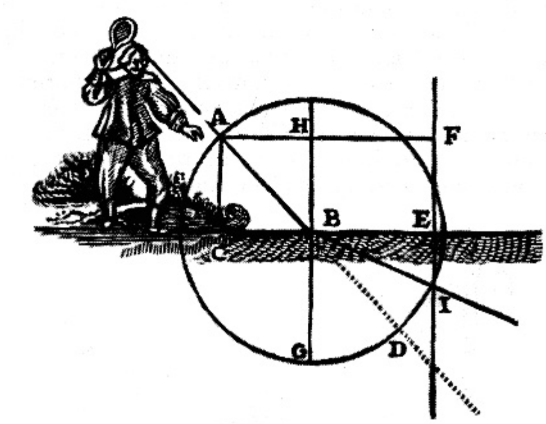


The law of the refraction according to Descartes.

In itself, the classic Cassegrain telescope possesses two essential virtues that can make it prefered to others by the amateur in search of a global system. The first known advantage resides in the possibility to use the primary for the deep sky. The second advantage will be to be equipped with various secondary of as many different magnifications as we could wish. No other telescope, Dall-Kirkham nor Ritchey-

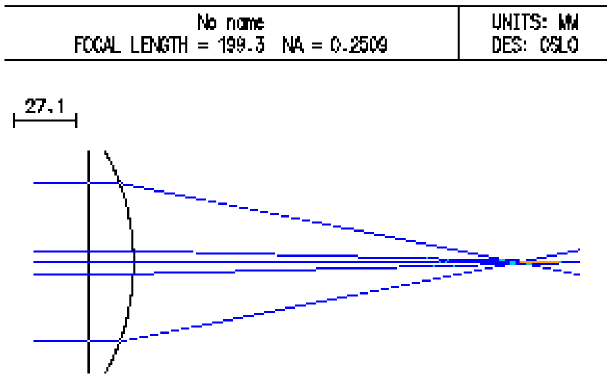
Chrétien,will possess such an interesting property for planetary seeing. As all telescopes of Cassegrain type, it is right to verify that the secondary present the wished conical constant. The solution seems evidently simple for the DK because the secondary is spherical. If one has a spherical caliber, it will be then easy to make fringes verification with a Fizeau interferometer, as proposed by Texereau in his well known work. Again we will have to achieve the aforesaid caliber to the wanted precision: double work for that. In the case of the classic Cassegrain or the RC, such a verification remains possible in the same way, trough the analysis of the fringes by a devoted software, as describes in my articles published on this site or in “L'Astronomie”[1]. In his *Dioptric* - a treaty relative to the study of the light refraction - Descartes introduces *the oval* that carries its name, and that are baptized also *aplanatic curve*. One will note here a common root with *planet*, the latin root being translated by mistake or error. An aplanatic curve is therefore a

[1] Juillet-Août 2007 & Octobre 2007.

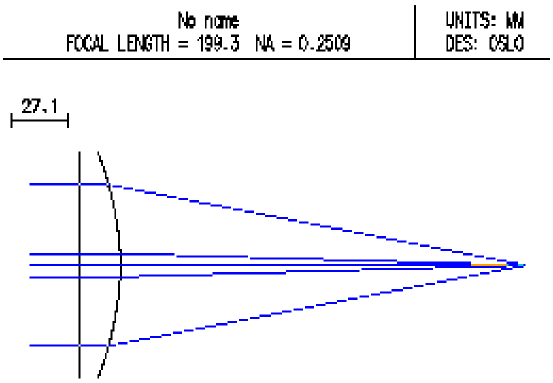
curve that is without *erring* or without *error*. We will see that if such a curve separates two medium of different indices, one can create a device where all incidental rays will converge in one unique point. In this primitive acceptance, *aplanatic* is here synonymous of *stigmatic*, from the Greek stigma, point[2]. Later this term will become synonymous of the absence of coma and spherical aberration in an optical system.

An Egg story

In fact the name of oval (from *ovo*, egg) is used here badly, the oval being a curve that draws itself by pieces. It is here an hyperbole with an eccentricity equal to the glass indices or if we prefer with a conical constant-b = n^2. Let us examine such a device under the arbitrary shape of a Plano-convex lens with a focal distance equal for instance to 200 mm, opened at F/D=2 and convexity turned to the focal plane. It presents a strong spherical aberration (see on the left). Now, give it a conical constant -b equal to the square of the glass index. One can now note that all rays will converge in only one point (see figure on the right). One makes here the simulation with a BK7 lens nd=1,518 at 587 nm, and -b=2,3. One sees the identity with the parabolic mirror.



Spherical lents: spherical aberration.



Hyperbolic lens, $-b=n^2=2,3$.

This interesting consideration paves the way to a test mode of the secondary mirror that will avoid the use of a caliber so long as one is able to make reflecting the plane face of the lens by a deposit of silvering for instance or an arranged mirror behind (but probably the 4% of reflection of the glass should be sufficient). The proposition of this method is due to Norman[3]. In those conditions, examined at the Foucault (that must be here done with nearly monochromatic light), it will appear with the characteristic flat tint of the sphere and examined with the Bath interferometer, the fringes will appear straight, the lens being used in auto collimation. Again it is necessary to know what we can make with a value of K=-2,3 for the secondary of a Cassegrain.

We know that this conical constant is linked in a simple way to the magnification G and doesn't depend on the radius of curvature for the classic Cassegrain:

$$-b_2 = \left[\frac{G+1}{G-1}\right]^2 ; (1)$$

2 http://serge.mehl.free.fr/anx/OvalDec.html

3 **Burt A.Norman**, *A New Test for the Cassegrain Telescope*, S&T novembre 1957, pp.38-40

Two solutions exist for -b2=2,3, the one that is satisfactory is G=4,87. In other words, all Cassegrain that will satisfie this double condition of magnification and conic, will see the control of its secondary possible *through the glass* that composes it, avoiding the use of a reference caliber, as far as this glass will have the required quality of homogeneity. For sure, a good quality glass is necessary even the volume is low. One can use a more elevated index glass, for example n_d=1,8 that will succeed to a magnification of 3,5 and a coefficient -b2=3,24. Quartz or silica present an index of 1,458, therefore G=5,38, with a very low temperature coefficient.

From that, one would be tempted to deduct that to change a testable secondary, the right way is to change the kind of glass. In fact it is not, with the help of a small trick about what we will speak latter. But let's have a look to what a Cassegrain gives with G=4,87. For that, let's calculate first the secondary to primary distance and then the secondary mirror ray of curvature of our combination, knowing we have a 200 mm rear distance, a primary diameter of 200mm with a F/D of 4,1 for the primary. One succeeds with the magnification of 4,87 in a F/D = 20, what is ok for a Cassegrain.

One finds a primary ray of -1642 mm, a secondary radius of curvature of -437,5 mm, a secondary primary separation of 647 mm and for ±0,3° of full light field, the diameter of the secondary will be of 50 mm, either an obstruction of 25%. Now, let us examine what one would see at the interferometer without and with the asphere.

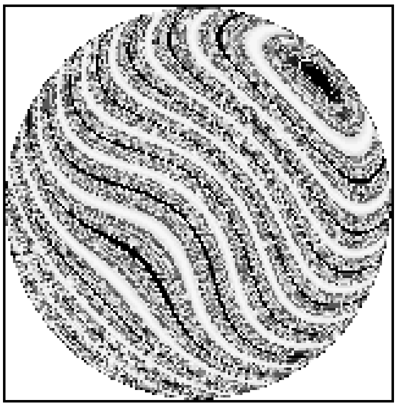
Conic

Gen | Setup | Wavelength | Variables | Draw Off | Group | Notes

Lens: No name — Efl -483.777927

Ent beam radius 25.000000 — Field angle 0.500000 — Primary wavln 0.587560

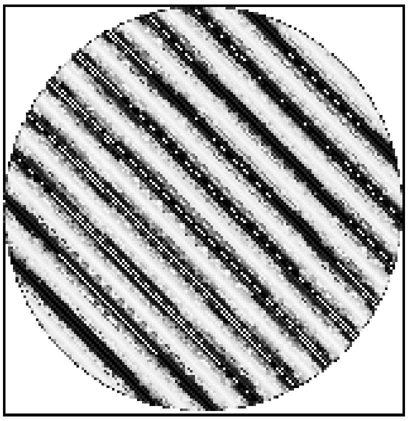
SRF	RADIUS		THICKNESS		APERTURE RADIUS		GLASS		SPECIAL
OBJ	0.000000		843.400000		7.360240		AIR		
AST	0.000000		0.000000		25.000000	AS	AIR		
2	437.000000		10.000000		25.000000	S	N-BK7	C	A
3	0.000000		100.000000		25.058040	S	AIR		
4	0.000000		-100.000000	P	70.000000		REFL_HATCH		
5	0.000000		-10.000000	P	26.818754	S	N-BK7	P	
6	437.000000	P	0.000000		26.876795	S	AIR		A
IMS	0.000000		-843.427581		7.493844	S			



P-V OPD
0.5964

RMS OPD
0.1778

Spherical, best focus.



P-V OPD
0.01241

RMS OPD
0.003584

With asphering, b2 = -2,3.

The distortion on the glass of raised P/V side type is given by:

$$\varepsilon = \frac{b}{32} \cdot \frac{h^4}{r^3} = \frac{2.3}{32} \cdot \frac{25^4}{437^3} = 0.336\mu \text{ ; (2)}$$

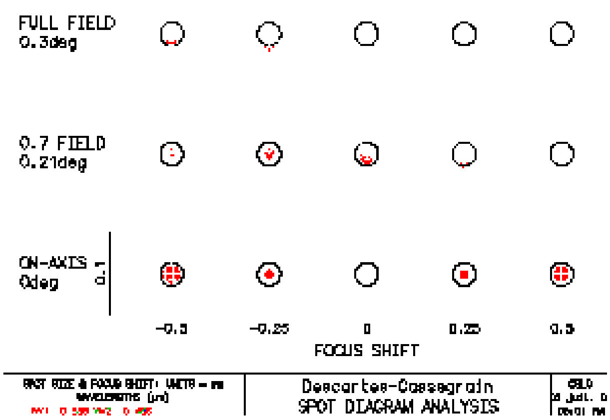
It can be achieved *in situ* under an interferometer control, with an annular tool, this is is very interesting in terms of development time and puts this type of Cassegrain directly in competition with the Dall-Kirkham, considering the absence of reference caliber to test and to correct with an unavoidable error.

What curvature can one admit on the flatness of the backplane of the secondary? Simulations show that the arrow measured with a spherometer should not exceed ±20µ on a 50 mm diameter to have lambda/15 to 587 nm, even on an ordinary floated glass. Now examine what such a Cassegrain will give compared to a Dall-Kirkham using the same optic elements with a F/20 opening.

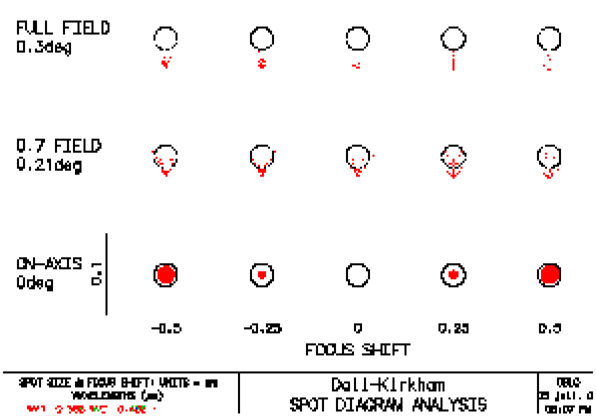
Gen	Setup	Wavelength	Variables	Draw Off	Group	Notes
Lens: Descartes-Cassegrain					Efl	4.0133e+03
Ent beam radius	100.000000	Field angle	0.300000	Primary wavln		0.587560

SRF	RADIUS	THICKNESS	APERTURE RADIUS		GLASS	SPECIAL
OBJ	0.000000	1.0000e+20	5.2360e+17		AIR	
AST	0.000000	0.000000	100.000000	AS	AIR	
2	-1.6420e+03	-647.000000	100.000000	S	REFL_HATCH	A
3	-437.500000	647.000000	24.581381	S	REFL_HATCH	A
IMS	0.000000	203.541059	21.013689	S		

Cassegrain telescope



Dall-Kirkham telescope

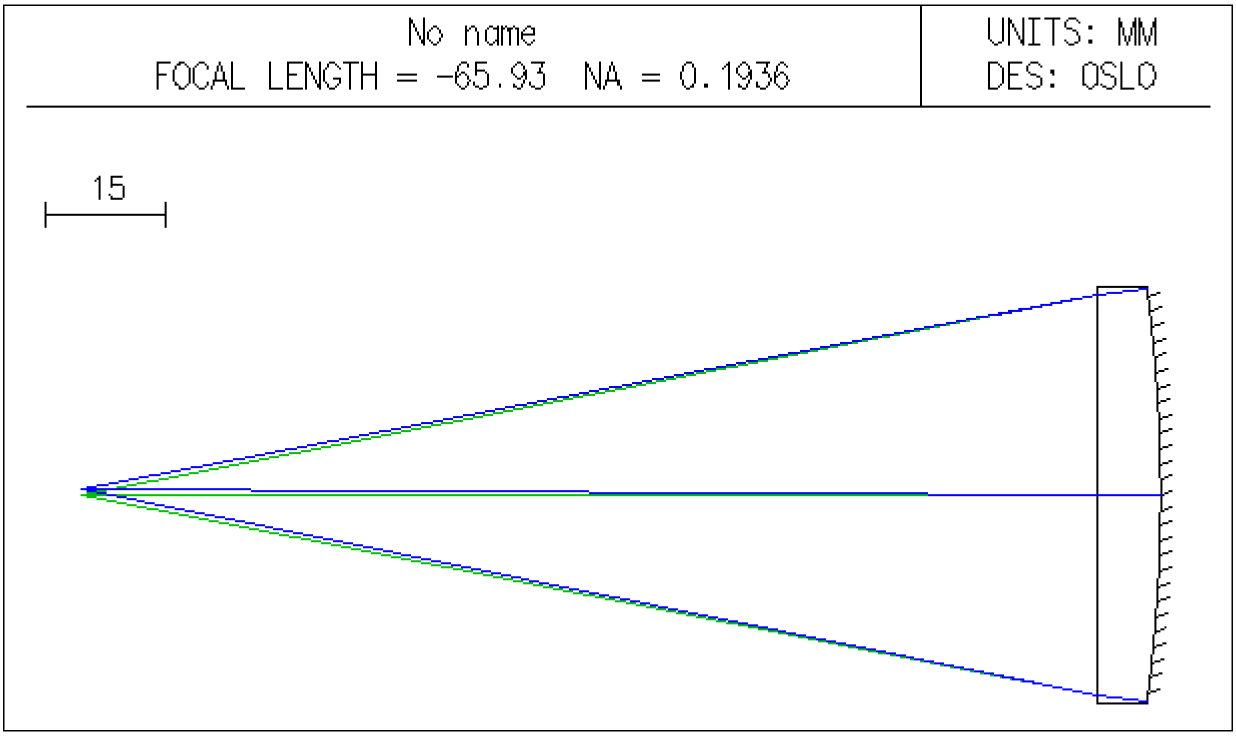


As we can see, the Cassegrain is usable on the totality of the ±0.3° field of the photograph field, dominated by the curvature and not by the coma on a diagonal of a 24x36 mm rectangle with a small -1/2 mm shift. That's not the case of the DK that would be certainly more at ease with a less open primary and an amplification G of only 3.5, at the cost of a stronger obstruction, harmful to the contrast. So necessary, the field can be yielded plan while adding a Plano-concave lens, put close to the focal plan in order to avoid position chroma.

It is nevertheless that a Cassegrain of which one cannot change G remained underused. Certainly, a x3 Barlow lens will permit to go to F/60 in ideal conditions for planetary in adeal conditions. But that is not this consideration that will originate us issue but rather the inverse question, what about a F/10 telescope, more deep sky capable, or even, what about a Cassegrain with a plane field where the curvature of the primary and the secondary are equal but opposed in signs? There the field reducers exist again. It introduce various aberrations that limit the exploitable field. Why not put the secondary on a bayonet in order to make it interchangeable? Again it is necessary to be able to test it in the same way… And if it is possible for a Cassegrain, this will also be for a Ritchey-Christian. It is going without saying that a solution exists…

Where the secondary is returned

A systematic mind won't miss to wonder what conditions must exist in the case where the reflection, instead of taking place on the plane face, would do so on the concave surface, inside the glass according to the figure below.



In such a case, the conic constant of the concave surface which eliminates the longitudinal spherical aberration is merely given by:

$$-b = n^2 - 1 \ ; (3)$$

In the case that occupies us, with nd = 1,517, one finds: -b = 1,304. What does G value corresponds to a conical constant of -1,304? There has two solutions again and the one that is usable is G= 15,27. With our primary opened to F/4.1, one will have F/D = 62,6. In these conditions, the radius of curvature of the secondary will be of -134,2 mm, his/her/its diameter of only 20 mm and the distance separating the two mirrors of 757,3 mm.

The distortion on the glass of type side raised P/V is equal to:

$$\varepsilon = \frac{b}{32}.\frac{h^4}{r^3} = \frac{1.3}{32}.\frac{10^4}{134.2^3} = 0.168\mu$$

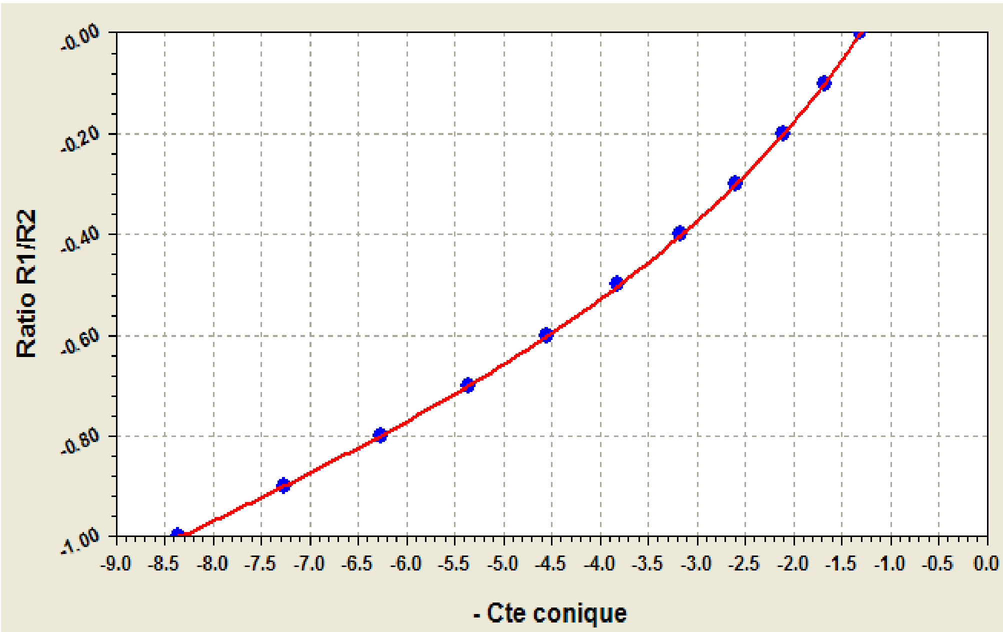
That's two times lower than previously but on a very small diameter, here 20 mm. Alas, the test should get used at 86 mm of the surface, what makes it unusable with the Bath interferometer but feasible probably with a Fizeau, or even a spherical Michelson or a Shack, the Foucault being *a priori* excluded.

What about instead having a plane face, one accepts to give it a certain curvature? One adds the spherical aberration that it will be necessary to compensate by a more elevated conical coefficient there. The relation that links the constant conical "b" and the R_1/R_2 ratio = *m* is given by the following expression[4]:

$$-b_2 = (1-m)^2.\left[n^2 - 1 - (n^2 - n)m\right] \; ; (4)$$

which is in fact a third degree equation when one develops it. For R_1/R_2= m= 0 (convex plan), one recovers the expression (3) and -b2 = -1.304, for m=-1 (bi convex lens) one finds-b2 = -8,36. One has there a large range of Conic Constant that can cover a large range of telescopes. That's being said, one is interested to know *m=f(-b2)*, because -b2 is directly spring on (1). Rather than to tempt an laborious inversion of the expression (4) a polynomial approximation of *m=f(-b)* will make as well the business. The free software, *Curve Expert,* has been used. With k= -b2, BK7 and Nd=587 nm:

$$m = 0{,}31 + 0{,}26.k + 0{,}012.k^2 \; ; (5)$$



[4] **Robert T.Holleran**, *Third Order Wavefronts and Related Null Test*, App. Opt., Vol.5, N°7, July 1966, p.1244.

A cassegrain plane field telescope.

We have seen that the field curvature was the weak point of the planetary Cassegrain. Now let us interest ourselves in a wide field Cassegrain with a plane field. For that, it is right to achieve the condition of equality of the curvatures of the mirrors, that the primary being concave and the secondary convex. What is the secondary conical constant and the distance of the two mirrors that will satisfy such a requirement?

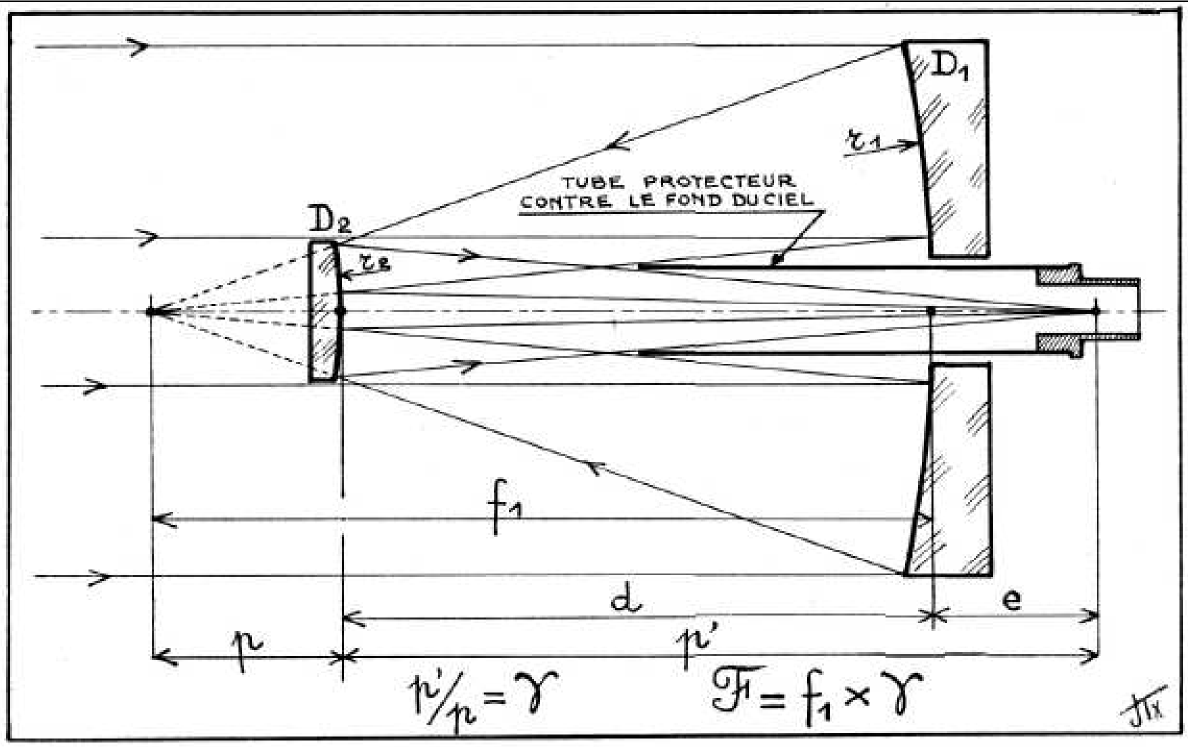


Fig. 70. – Schéma général d'une combinaison Cassegrain.
Les dimensions transversales sont exagérées.

However we know according to Texereau

$$r_2 = \frac{2.p.G}{G-1} \;;\; (6)$$

$$p = \frac{f_1 + e}{G+1} \;;\; (7)$$

And also that r2 = $2.f_1$. While combining the expressions (6) and (7), one finds then:

$$\frac{G}{G^2-1} = \frac{f_1}{f_1+e} \;;\; (8)$$

It is interesting to note that when e = 0, the solutions of this second degree equation the solutions are the gold number: 1.618 and -0.618, the first solution corresponding to the Cassegrain telescope.

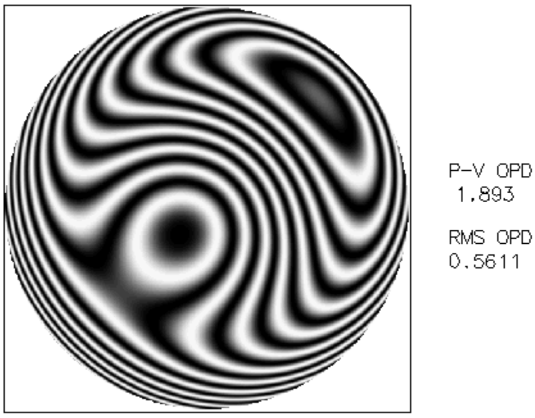
Let us take our 200 mm telescope. It remains to calculate only G, the conical constant-*b2 & p*. One finds G = 1.8, - *b2* = -12.25, *p* = 364.4 mm, *p* ' = 656 mm and therefore *d* = 656-200 = 456 mm. In these conditions, *r2* = 1640 mm (that is evidently the primary radius of curvature) and the opening will be of F/7,2. The diameter of the secondary for a field of full light equals to null, will be 89 mm and the obstruction, 44%; for a ±0,3° field of is ½ inch, the obstruction goes up to 46%. A primary opened at F/3 or even to F/3,5 would be a better inspiration or then, to tolerate an F/10 opening and a field slightly curved... Have a look nevertheless what such an instrument will give because the goal here is to show the feasibility of the test of the secondary. The calculation of the expression (5) gives us -1.07. It is there about an approximation outside of the limit of validity, in fact we are close to -1,3 as one can find it with the expression (4). Therefore, the ray of curvature of the front face is: R1 = 1640/-1,3 = -1261 mm. Simulation tests confirm the result.

Gen | Setup | Wavelength | Variables | Draw Off | Group | Notes

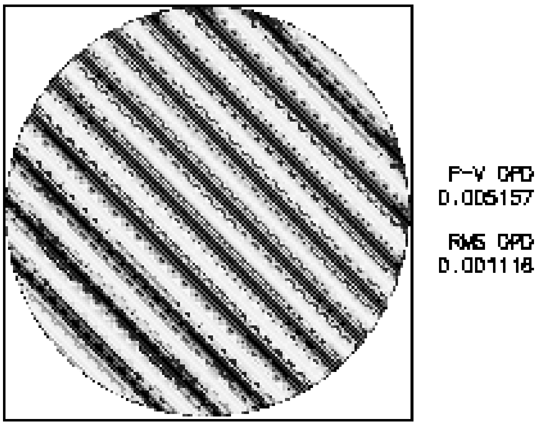
Lens: No name — Efl -375.471497

Ent beam radius 45.000000 — Field angle 1.000000 — Primary wavln 0.587560

SRF	RADIUS		THICKNESS		APERTURE RADIUS		GLASS		SPECIAL
OBJ	0.000000		747.000000		13.038934		AIR		
AST	0.000000		0.000000		45.000000	AS	AIR		
2	1.2610e+03		10.000000		45.000000	S	N-BK7	C	
3	-1.6400e+03		0.000000		25.000000		REFL_HATCH		
4	1.2610e+03	P	0.000000		45.390648	S	AIR		
IMS	0.000000		-748.285314		12.981873	S			



Without distortion of the backplane.



With k=-12,25.

It is now useless to convey on the rest of this telescope or its feasibility, the objective being to show that the null-test of the secondary calculated in a particular context, is functionnal.

To Conclude

For the first time, we have synthesized here little known, scattered or forgotten considerations, to show that the realization of hyperbolic secondaries and especially their tests were extensively accessible to the amateur so long as he uses for the secondary, optical quality glass.

Contrarily to a widespread opinion, which I have to confess I only shared too long, the realization of a Cassegrain secondary is probably not such a long and fastidious task to the point to prefer a spherical secondary, at least in a particular case, the one in which the conical constant is fixed equal *a priori* to the square of the indices of the material of the aforesaid secondary. In this precise case, because the rear surface is plane, one has a very interesting null-test that permits to have precise criteria to stop the work of the glass distortion: right fringes or a flat hue at the Foucault knife.

But there is not the only merit of this method exposed about half a century ago by Burt A. Norman† in S&T and Robert T. Holleran and whose names deserves to be kept, or even to be attached to this kind of test or telescope.

The second advantage which it is right to recall, is the absence of a caliber or any reference that it would be necessary to achieve in addition to the secondary; then the final adjustment of the conic can be achieved without leaving the measure stand of the interferometer or the Foucault. This nearly absence of manipulation can only be beneficial to precision and quality of work. Finally, in spite of its frozen value in the simplest case, the conic coefficient well corresponds to what one is waiting for a Cassegrain.

We also showed that if one abandons the plane face, one can reach whichever conical constants at the cost of a small extra work. It is right to apply the expression (5) in its validity limit and to verify it with expression (4). An example has been shown with a very elevated CC, but the process is in any case, the same. An ultimate verification with Oslo will be a plus. This method becomes then usable for all types of telescopes.

This doesn't evidently mean that the DK loses all its existential reasons, because it remains superior to the classic Cassegrain in terms of stability and collimation easiness, what can be an important criterion for planetary work. But it is clear that this method, derived from Cartesian considerations, is very tempting.

Charles Rydel, Dec 7 2008.