High resolution imaging of the solar system

Basic rules: resolution and optimal sampling

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Introduction

High resolution... who doesn't dream of achieving it, of getting images of the Moon, the Sun and the planets full of fine details?

Achieving high resolution means obtaining the finest detail that the instrument is capable of delivering. This notion therefore depends on the instrument and the camera, it is not absolute.

The formulas below explain how to find the right telescope-barlow combination, depending on the pixel size of your camera, to achieve high resolution.

These formulas are useful but not sufficient: your optics must also be well **collimated** and the sky must cooperate with minimal turbulence. And speaking of collimation: experience has shown me that with a Schmidt-Cassegrain or a Newtonian telescope, to hope to reach high resolution, you have to *collimate the optics before each session, even if the instrument is permanently mounted!*

Some basic notions, some formulas...

The **optical resolution** of a telescope is the smallest apparent angle between two points that can be told apart by this instrument. Optical resolution is expressed in **arcseconds**. An arcsecond is the 3600th part of a degree. The resolution is calculated as[1]:

$R = 120/\mathrm{D}$ eq. 1

where D = diameter of the instrument in mm

For example, an 8-inch (203 mm) diameter telescope provides an optical resolution equal to R = 120/203 = 0.59 arc second.

High resolution imaging means obtaining an image in which two points can be distinguished from each other when they are separated by an angle corresponding to the resolution of the telescope. For example, if two lunar craters are 0.59 arcseconds apart and the telescope has a diameter of 203mm, an image taken through the telescope will qualify as high resolution if the two craters are separated. If the two craters appear as an elongated crater or an indistinct spot, then the full potential of the optical system has not been exploited.

The aim is to obtain a **photographic resolution** at least equal to the optical resolution. To achieve this, it is necessary, for a digital camera, that two details whose apparent angle is that corresponding to the optical resolution of the telescope fall on two different pixels. This brings us to the notion of **sampling**.

Sampling is the amount of sky covered by **one** camera **pixel.** It is measured in **arc seconds per pixel**. The information (number of photons, "grains" of light) that falls on a pixel is measured and translated into a number of electrons. This number of electrons is coded with varying degrees of accuracy (number of bits per pixel). The sampling is calculated as follows:

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[1] This is an approximate formula, accurate for blue light (about 480 nm wavelength). The resolution improves when the wavelength shortens, but on the other hand short wavelengths are more sensitive to bad seeing.

$E = \frac{206 \times pix}{F}$ where *pix is* the width of a pixel in microns and *F is* the focal length of the telescope in mm equ. 2

For example, a telescope with a focal length of 2030 mm (such as a C8 at F/D=10) and a camera with pixels of 3.75 microns (such as the ASI224MC) gives a sampling equal to

$E = \frac{206 \times 3,75}{2030} = 0,38$ arcsecond per pixel.

Details falling on a single pixel are not resolved (separated) on the image. Therefore, in our example, details 0.38 arcseconds apart are not resolved on the image.

To achieve high resolution, two details whose angular distance corresponds to the resolution of the telescope must fall (on average) on two different pixels. It follows that the sampling must be at least two times finer than the resolution of the instrument:

$$E = \frac{R}{2} = \frac{120}{2 \times D} = \frac{60}{D} \qquad \text{equ. 3}$$

However, equation 3 assumes that the two details to be resolved fall on two pixels placed side by side. However, there is no guarantee that this will be the case for all the details (on the contrary). The details to be resolved must fall on *two diagonally placed pixels*. In other words, two details whose distance is that of the telescope's resolution must be projected onto the sensor at a distance corresponding to 2 × the pixel size × $\sqrt{2}$. The sampling must therefore be:

$$E = \frac{R}{2\times\sqrt{2}} = \frac{120}{2\times\sqrt{2}\times D} = \frac{60}{\sqrt{2}\times D} \qquad \text{equ. 4}$$

For example, to achieve high resolution with an 8-inch Celestron, you need to achieve a sampling rate equal to $E = \frac{60}{\sqrt{2}\times 203} = 0.21$ arc second per pixel.

This value gives a **photographic resolution** equal to the optical resolution *R*, i.e. 0.21 × 2 × $\sqrt{2}$ = 0.59 arc second.

Finally, for the algebraically inclined, we can combine equations 1, 2 and 3 and show that we can directly calculate the F/D ratio for high resolution on the basis of pixel size alone, by positing:

$$R = \frac{120}{D} \quad E = \frac{206 \times pix}{F} = \frac{60}{\sqrt{2}\times D}$$

Therefore, by isolating F/D:

$$\frac{F}{D} = \frac{\sqrt{2} \times 206 \times pix}{60} = 4,86 \times pix \approx 5 \times pix \qquad \text{equ. 5}$$

This ratio is valid regardless of the diameter of your instrument. [2]

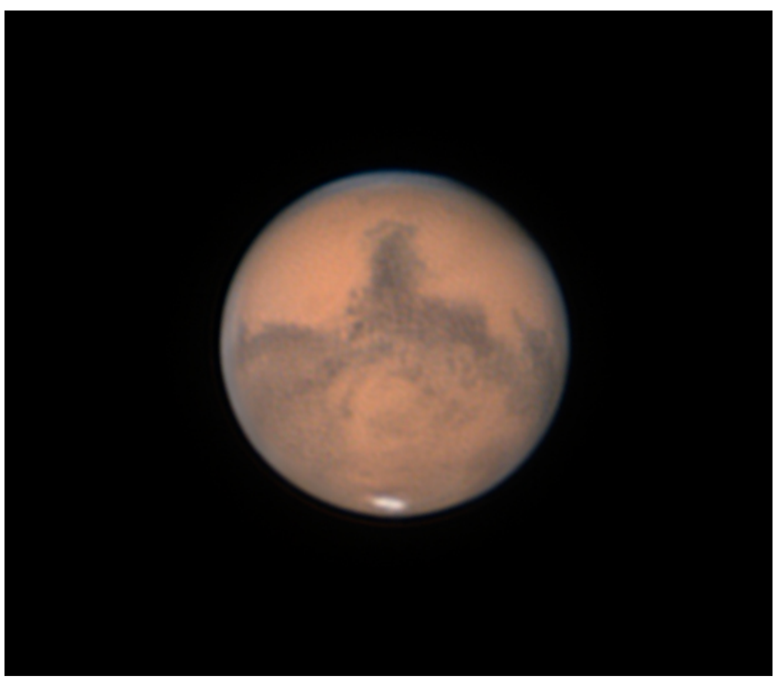
Example: to achieve high resolution with a camera whose pixels are 3.75 microns square, you need an F/D ratio of 5 × 3.75 = 18.75. If you have a telescope open at F/D=10 (like most SCTs), a 2× barlow

[2] Another way of looking at it gives almost the same result: you want to have three pixels in a row to ensure maximum resolution; in other words, sampling equal to one third of the resolution of the instrument. *E is* then equal to 40/D. The *F/D* ratio to be achieved is therefore equal to (206 × *pix*)/40 = 5.15 × *pix* ≈ 5 × *pix*.

will suffice to achieve high resolution (if the quality of the optics, collimation and turbulence allow it).

That said, why not magnify even more to get a bigger image on the sensor? We are often tempted to magnify to the maximum in the hope of imaging more details, or simply to have a bigger image on the screen. This is usually an illusion... indeed, except for the rarest situations (perfect seeing, near perfect optics), which allow a smoother image when working a little further from the limit, oversampling) is not only useless, but harmful. Why is this? For two reasons. The first is that the instrument is already at the maximum of its capacity. A higher magnification does not reveal more detail. And secondly, *because each additional magnification darkens the image*, so you have to expose longer (e.g. 5 milliseconds instead of 2) or increase the gain of the camera. A longer exposure is more likely to result in a blurred image due to turbulence. So, not only can we not resolve more details because we are already at the limit of what the telescope can resolve, but we also lose some because of too long exposure! Thus, going from F/D=10 to F/D=20 (with a 2× barlow) requires you to pose 4 times longer. And if F/D=20 allows you to have high resolution, going to F/D=30 (e.g. with a 3× barlow instead of the 2× barlow) will force you to more than double your exposure time without any compensation. Or, if you want to maintain the exposure time, you will have to increase the gain of the camera, which will produce grainier images... which, even combined by the tens of thousands, will not reveal more detail than your optics allow. So, whenever the urge arises to magnify even more, remember that you will be wasting light, and that the camera, which cannot violate the laws of optics, will not reveal details that the telescope cannot!

Finally, remember that these rules are valid for **solar system imaging only**. Deep sky images, which usually require exposure times of several minutes, cannot achieve such resolutions because turbulence inevitably drowns out detail. In the latter case, it is more than sufficient to aim at an optimal sampling to resolve details corresponding to the *average* turbulence of the site. For example, if your site has an average seeing of 2.6 arc seconds (as is often the case in Quebec), a sampling of 1 arc second per pixel will be sufficient.



Mars, 6 October 2020 03h17 UTC
ONTC1010 Newtonian telescope, 254 mm F/4, Astro_Physics 1200 GTO
Televue PowerMate 5x, ZWO ASI224MC, Astronomik IR-cut filter