Introduction to Fermi acceleration

Denis Gialis*

Fermi acceleration is one of the most efficient acceleration process for particles in the Universe. It was first proposed by Enrico Fermi in 1949. This process occurs in many different astrophysical situations like - e.g. - solar corona, interstellar clouds, supernovae, Wolf-Rayet stars, pulsar winds, Gamma-Ray Bursts, microquasars or Active Galactic Nuclei... In this short introduction, I intend to explain some physical basis of such a process in a general context.

1 The non-relativistic case

In this first section, we consider "non-relativistic" Fermi acceleration of particules. In the medium (or plasma), we neglect binary interactions and particles only collide or interact with some scattering centers of any origin. In astrophysical contexts, their nature is often magnetic (MHD disturbances, magnetic clouds...). These scattering centers are supposed to have an infinite mass and a non-relativistic velocity $\vec{u}$, in the observer frame: the "non-relativistic" character of the considered process refers to this velocity. For simplicity, we assume that all interactions can be considered as elastic i.e the shock or the interaction, between the particle and the scattering center, does not absorb a significant energy. Initial particle velocity, namely $\vec{v}_i$, is such that $v_i \ll c$, in the observer frame. This assumption is not essential but it simplifies the explanation below, allowing galilean velocity transformation. Following these assumptions, in a scattering center frame, the kinetic energy of a particle is conserved during the interaction. But, in the observer frame, the situation is quite different. For a particle, one can write

$$\Delta\epsilon_c = \epsilon_{cf} - \epsilon_{ci} = \frac{1}{2}\, m\, \left(v_f^2 - v_i^2\right)\ , \tag{1}$$

*Ph. D. in Astrophysics - University J. Fourier, Grenoble, FRANCE.

where v_f is the particle velocity, in the observer frame, after the interaction. In the scattering center frame, velocities before and after the interaction are

$$\vec{v}_{ci} = \vec{v}_i - \vec{u}\,, \tag{2}$$
$$\vec{v}_{cf} = \vec{v}_f - \vec{u}\,, \tag{3}$$

and, because particle does not gain energy in reflection, we have, in first approximation: $\vec{v}_{cf} = -\vec{v}_{ci}$. Thus, the energy conservation, in this frame, leads to

$$\Delta\epsilon_c = m\,\left(\vec{v}_f - \vec{v}_i\right)\cdot\vec{u} = 2m\,\left(u^2 - \vec{v}_i\cdot\vec{u}\right)\,. \tag{4}$$

For a head-on interaction, $-\vec{v}_i\cdot\vec{u} > 0$, and the energy gain, $\epsilon_{ci}/\epsilon_{cf}$, is always > 1. For a rear-on or overtaking interaction, the energy gain can be < 1 if $|\vec{v}_i\cdot\vec{u}| > u^2$.
The efficiency of such a process, like the characteristic acceleration time, highly depends on the considered astrophysical situation, and its determination has to take account of several limitations (synchrotron losses, binary interactions, ...). By neglecting these limitations, we can distinguish two important cases:
For an equivalent high number of head-on and rear-on interactions, the gain coming from the term $\vec{v}_i\cdot\vec{u}$ will be close to 1. But the term $2m\,u^2$ will be always > 0, and particle energy could increase at every interaction. This is the *stochastic second order (in u^2) Fermi process*.
However, when the scattering centers have an isotropic velocity distribution, the number of head-on collisions is higher than rear-on collisions, and thus, the energy gain, coming from the scalar product $\vec{v}_i\cdot\vec{u}$, is > 1. This is the *first order Fermi process*.
At last, following the physical context, characteristic acceleration times of these two Fermi processes could be quite different or not, and we cannot easily decide which one is the most efficient for particle acceleration.

2 Relativistic Fermi acceleration

We consider now a physical situation in which the scattering centers have a relativistic velocity. So we cannot use galilean transformations from a frame to another, but rather Lorentz transformations. In this way, we show here that the previous results are deeply modified.
Let be the 4-momentum, $(\epsilon_1, \vec{p}_1)$, of a particle in the observer frame. This particle interact with a scattering center, the velocity of which is $\vec{\beta}_\star$ $(= \vec{v}_\star/c)$ for the observer. Let be the pitch angle cosine, namely $\mu_1 = \cos(\vec{p}_1, \vec{\beta}_\star)$, of this interaction. We can define $\vec{p}_1 = \vec{p}_{1\|} + \vec{p}_{1\perp}$, where $\|$ and $\perp$ are relative to the $\vec{\beta}_\star$ direction, $p_1 = |\vec{p}_1|$ and $p_{1\|} = p_1\,\mu_1$. Lorentz transformation from the observer frame to the scattering center frame only modifies the momentum component $\vec{p}_{1\|}$. The 4-momentum $(\epsilon'_1, \vec{p}\,'_1)$, in the scattering center frame, is such that

$$\begin{aligned} \epsilon'_1 &= \gamma_\star \left(\epsilon_1 - \beta_\star\, p_{1\|}\right) , \\ p'_{1\|} &= \gamma_\star \left(p_{1\|} - \beta_\star\, \epsilon_1\right) , \end{aligned} \tag{5}$$

with $\beta_\star = |\vec{\beta}_\star|$ and $\gamma_\star = (1 - \beta_\star^2)^{-1/2}$.
Considering only relativistic particles, we have $\epsilon_1 \simeq p_1\, c$, and the previous equations yields

$$\begin{aligned} p'_1 &\simeq \gamma_\star \left(1 - \beta_\star\, \mu_1\right) p_1 , \\ \mu'_1 &\simeq \frac{\mu_1 - \beta_\star}{1 - \beta_\star\, \mu_1} . \end{aligned} \tag{6}$$

During the interaction, the particle energy is conserved in the scattering center frame: $p'_1 = p'_2$, the momentum after the interaction, and μ'_1 is transformed in μ'_2. A new Lorentz transformation gives the new 4-momentum, $(\epsilon_2, \vec{p}_2)$, of the particle with:

$$\begin{aligned} p_2 &\simeq \gamma_\star^2 \left(1 + \beta_\star\, \mu'_2\right) \left(1 - \beta_\star\, \mu_1\right) p_1, \\ \mu_2 &\simeq \frac{\mu'_2 + \beta_\star}{1 + \beta_\star\, \mu'_2} . \end{aligned} \tag{7}$$

When the pitch angle is $> 1/\gamma_\star$, the energy gain could be as high as $\gamma_\star^2$ in only one interaction: this implies that one cannot use non-relativistic Fokker-Planck description for the evolution of the phase-space particle distribution, because of the hypothesis of small energy jump at every interaction. In our case, the energy jump can be easily written:

$$\Delta\epsilon = \epsilon_2 - \epsilon_1 \simeq \beta_\star \, \frac{\mu_2 - \mu_1}{1 - \beta_\star \, \mu_2} \, \epsilon_1 \, . \tag{8}$$

When one consider interaction between particle and magnetic disturbance, μ'_1 is randomly transformed in μ'_2: thus, following Eq. (7), a flat distribution of the pitch angle cosine μ'_2 leads to an highly anisotropic distribution of μ_2 for $\beta_\star$ very close to 1, which is an important consequence of the relativistic Fermi acceleration.