Secrets of Telescope Resolution

Computer modeling and mathematical analysis shed light on instrumental limits to angular resolution. By Daniel W. Rickey





EVEN ON A GOOD NIGHT, the view of Jupiter in my telescope is never perfect. But besides the blurring effects of the atmosphere and the shortcomings of my own eyes, what causes the image to lose sharpness? Specifically, how do aperture, central-obstruction size, optical misalignment, and defocusing affect the view? By using computer modeling we can ignore certain variables and concentrate on ones that are inherent to the instrument — especially those we can do something about.

Spatial Resolution

To determine how various factors affect a telescope's ability to resolve fine detail (its *spatial resolution*) it is useful to turn to linear-systems theory. This mathematical tool deals with the frequency content of an image, which is related to the amount of detail present at various angular scales. Just as high (or low) radio frequencies correspond to short (or long) wavelengths, so do high (or low) spatial frequencies correspond to fine-scale (or large-scale) details in an image. By way of example, consider the images of Jupiter at the top of the facing page. The one at top left is the original, which contains a wide range of spatial frequencies. The

Effects of Aperture

To see how linear-systems theory works in the real world, let's start with the most basic telescope parameter: aperture. Something that observers often ponder is how big a telescope is needed to see planetary details such as Jupiter's Red Spot or the Martian polar caps. To answer this question look at the diagram at left, which shows MTF plots for four unobstructed telescopes of different apertures. The MTF graph plots how well image contrast is reproduced as a function of spatial frequency. Plots extending to high frequencies (toward the right) indicate telescopes that produce images with finer detail. For any given frequency, the higher the curve, the better the image contrast. (It is important to note that magnification does not affect resolution as expressed this way.) The graph for a perfect telescope of infinite aperture would simply be a horizontal line with a modulation transfer equal to 1.0 at all frequencies.

A telescope must be able to pass frequencies up to approximately 0.2 cycles per arcsecond — that is, to resolve details as small as 5 arcseconds — to produce an agreeable image of Jupiter's equatorial belts. As the plot shows, the modulation transfer for a 25-millimeter (1-inch) scope is only 0.05 (5 percent) at a frequency of 0.2 cycles per arcsecond. Thus we would expect Juview at top right looks like something we might see through a telescope on a very bad night — it contains only low spatial frequencies (large-scale features). The lower two images have been filtered to show only their mid-range and highfrequency components and obviously don't relate directly to the eyepiece view. Nevertheless, these illustrations provide a useful means of revealing the spatial frequencies that make up a telescopic image.

Linear-systems theory also makes use of the concept of *modulation transfer*, which is illustrated below. A modulation transfer of 1.0 means that a telescope reproduces the original view perfectly. The illustration shows that as

 Normal
 Low frequencies

 Mid frequencies
 High frequencies

This series of photos illustrates the spatial frequency content of an image of Jupiter. (The rings surrounding the planet are artifacts of the imaging process.) points to consider. First, contrast and resolution are inherently coupled; a telescope that does a poor job of preserving image contrast will be unable to resolve certain details. Second, a telescope is, in effect, a spatial filter – light enters the instrument and emerges as a filtered image. For example, a "filter" (be it defocusing, poor collimation, or some other aberration) that removes a lot of highfrequency information results in a blurry image. The mathematical relations that describe the image-forming process and how it affects spatial resolution are the modulation-transfer function (MTF) and its close relative, the system-characteristic function (SCF). I wrote software to compute these functions for various telescope de-

the modulation-transfer value decreases, the contrast in the image decreases until the modulation transfer equals zero and we see nothing.

With these concepts in mind, there are two important

signs based on formulas in the classic text *Principles of Optics* by Max Born and Emil Wolf. I soon found that by thinking of telescopes as spatial filters governed by MTFs and SCFs, their imaging properties become easier to understand.



piter's thick belts to be heavily smoothed with poor contrast and the thinner belts to be invisible in such a tiny instrument. This is illustrated at far right, where the affects of aperture have been simulated on images of Jupiter. As expected, images produced by 25- and 50-mm telescopes lack detail. The 200-mm scope, on the other hand, is in theory capable of producing a very high-resolution image, though in practice it will be limited by other factors such as atmospheric seeing.

Although the MTF graphs do a very nice job of showing how larger apertures provide better spatial resolution, they don't account for the increased light-gathering power of larger scopes because they're always normalized to a value of 1.0 at zero frequency. So instead we turn to the system-characteristic function. The differences at zero frequency are directly related to differences in aperture area. The light-collection and spatial-resolution advantages of the 200-mm scope compared with the smaller instruments are now quite dramatic. This is apparent not only in the graph, but also at the eyepiece. In many respects, the SCF more accurately depicts the performance differences between scopes than does the MTF.



This plot shows the system-characteristic functions (SCFs) of three unobstructed telescopes and takes into account differences in light grasp. Compare it with the MTF plot on the facing page.

Obstructed Apertures

The results shown in the previous section applies to unobstructed telescopes, such as refractors. Of course, many amateur astronomers use Newtonian reflectors or catadioptric telescopes. These have secondary mirrors 25 to 40 percent the diameter of their primary mirrors. An obstruction affects a telescope's resolution, but how exactly? The answer is complex and a frequent source of misunderstanding among telescope makers and users.

First, consider the MTF plots corresponding to various obstructions, below. These show that an obstruction will reduce image contrast at middle frequencies corresponding to scales of a few arcseconds — where a lot of planetary detail is found. Rather unexpectedly, it appears that the obstruction has increased image contrast at very high frequencies. This is misleading because we have not considered the amount of light lost due to the obstruction.



A better understanding is given by the SCFs plotted below, which clearly show that the obstruction has not improved image contrast at high frequencies. Instead, image contrast at low to mid frequencies has been significantly reduced. The effect is simulated in the pair of Jupiter images. A large obstruction reduces the overall image brightness as well as the contrast of the planet's cloud belts. A large obstruction is generally not a good thing.



The effect of a central obstruction is one of the most frequently debated topics among amateurs. The simulated views of Jupiter show the impact of a 40-percent obstruction (bottom image) compared with the view through an unobstructed scope. As the accompanying SCF graph shows, the effect of a 25-percent obstruction is much less.

Mechanical Issues

Observers often underestimate the importance of focus and collimation with respect to telescope performance. The focusers supplied with many telescopes are too coarse and have too much mechanical play for accurate focusing. But how good does your focuser need to be? Plotted at right are the MTF curves for different amounts of defocusing. It's pretty obvious that with an f/5.6 telescope, even as little as 0.04 mm of defocusing has a significant impact on the MTF. To further illustrate this effect, corresponding images are also shown. With as little as a 0.06-mm eyepiece displacement from perfect focus, the image is noticeably blurry. For this telescope, ideally we want accurate and precise control to about 0.03 mm, which requires a very good focuser. To estimate (in millimeters) the focusing accuracy needed for your scope, multiply the square of your scope's f/ratio by 0.001. Thus, if you have an f/15 telescope, you want a focuser with a precision of about 0.2 mm ($15^2 \times 0.001$).

The most common maintenance most telescopes require is collimation. Unfortunately, the adjustments provided are often difficult to use and lack the necessary precision. Shown at far right are MTF curves for various degrees of optical misalignment in an f/5.6 Newtonian. The corresponding images of Saturn convey the effects of poor collimation. These illustrations make it apparent that this scope can tolerate a misalignment of no more than a few arcminutes.

It's important to note that the difficulty of achieving and maintaining collimation increases quickly with decreasing focal ratio. For example, if you use a laser collimator to align the optics of a 200-mm f/4 reflector, the returning beam of

Final Thoughts

The analysis and sample images in the colored boxes show that MTF and SCF plots are useful tools for understanding the individual consequences of common telescope problems. But, unsurprisingly, reality is more complicated. Most telescopes suffer to some degree from a combination of all of these aberrations and, as their effects accumulate, produce images that increasingly lack sharpness and contrast. This is demonstrated in the images of Jupiter at right, where the effects of obstruction, misalignment, and defocusing are added sequentially. Though no individual problem completely ruins the view, all three in combination produce results that are terrible. The lesson here is that you have to be mindful of *all* sources of image degradation. For example, a scope that can't be accurately collimated is going to yield poor views even if it is properly focused.

Although detailed analysis of MTF and SCF plots can tell us a great deal, there are obviously other factors to consider when selecting a real telescope. In general a larger aperture is better than a smaller one, though other factors (such as optical quality and the size of one's bank account) quickly become important. A small secondary obstruction is preferred, but its size is often dictated by the telescope's overall design, so there may not be much choice in this matter.



As these simulated images of Saturn and this MTF plot for a 200-mm f/5.6 telescope show, precise focusing is essential for detailed views (the amount of defocusing is noted next to each image). For an f/4 instrument, displacement values half as great produce the same MTF curves and views of Saturn.

light must fall within 0.35 mm of its target, which is much smaller than the central hole in most collimators. However, for a 200-mm f/8 instrument, the returning laser beam must be within a relatively achievable 2.8 mm of center. Obviously a laser collimator alone isn't going to be enough for a fast telescope — the alignment must be fine-tuned by star testing. Importantly, a low-quality focuser that allows the eyepiece or camera to droop will also affect the collimation, and, as a result, the quality of the image.

In the real world. telescope problems don't happen in isolation from one another - poor images are often the result of a combination of errors. These simulated views of Jupiter show how bad it can get if an obstructed telescope is poorly collimated and not properly focused.



But it is worth noting that the effects of an obstruction are minor compared to those resulting from poor focusing and miscollimation, as Damian Peach's superbly detailed Mars image (at right) so forcefully demonstrates. One can always add a high-quality after-market focuser, and by choosing a scope of moderate f/ratio or by using a well-designed mirror cell, you can increase the likelihood that precise and repeatable collimation will be achievable. Indeed, Accurately collimating your telescope is one of the quickest and easiest ways to improve its images. As the MTF plot for a 200-mm f/5.6 telescope shows, only a few arcminutes of misalignment is necessary to blur the image of Saturn. To apply these curves to an f/8 instrument, double the misalignment values; for an f/4 scope, halve them.



When everything is working right, the reward is a stunning image like this one of Mars, obtained last November by Damian Peach with a 14-inch Celestron Schmidt-Cassegrain telescope. The scope's large aperture combined with precise focusing and collimation more than make up for the effects of its 32% central obstruction.

because the tolerances for alignment and focusing become increasingly strict as the focal ratio decreases, perhaps the underlying message of this analysis is that telescopes with moderate f/ratios remain a wise choice. *

DANIEL W. RICKEY is a medical physicist at CancerCare Manitoba in Canada and spends his days working with ultrasound and magnetic-resonance-imaging scanners. Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.