Variability of the solar shape (before space dedicated missions)

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A B S T R A C T

Shrinking or expansion of the solar shape and irradiance variations are ultimately related to solar activity. We give here a review on existing ground-based or space solar radius measurements, extending the concept to shape changes. We show how helioseismology results allow us to look at the variations below the surface, where changes are not uniform, putting in evidence a new shallow layer, the leptocline, which is the seat of solar asphericities, radius variations with the 11-yr cycle and the cradle of complex physical processes: partial ionization of the light elements, opacities changes, superadiabaticity, strong gradient of rotation and pressure. Based on such physical grounds, we show why it is important to get accurate measurements from scheduled dedicated space missions: PICARD, SDO, DynaMICCS, ASTROMETRIA, SPHERIS. Such measurements will provide us a unique opportunity to study in detail the relationship between global solar properties and changes in the Sun’s interior.

1. Introduction

Since the highest Antiquity the determination of the value of the solar diameter has been a subject widely debated. A number of historical books already treated this question and the topic could be considered as ended. By opening a book on Astronomy, such as the Astrophysical Quantities (Allen, 2000), one may find the value

\[ R_\odot = 6.955\,08 \pm 0.00026 \times 10^8 \, \text{m} \]

which appears as a definitive measure of the solar radius and is thus widely used today. However, looking carefully at the question, it is not so obvious. First, an absolute value is not yet determined. Just as an example, a discussion of measurements of the solar diameter made during the XIX century by Wittmann in several papers (see for instance Wittman, 1980; Wittmann and Bianda, 2000) yields \( R_\odot = 696.265 \pm 65 \) km (with no temporal trend within the accuracy of the measurements), whereas measurements made by these authors at Izaña during the years 1990–2000 yields \( 960.63 \pm 0.02 \) (arc second, at 1 AU), always without any significant cycle-dependence variations in excess of about 400 km (Wittmann and Bianda, 2000; Wittmann et al., 1981). Such values are different from that adopted by Allen (2000). Moreover, giving a value of the solar diameter requires a definition, as the Sun is not a spherical solid. Several expressions can be given. The most commonly accepted is the diameter defined as the distance taken between the two opposite inflection points of the limb intensity profile, at a given wavelength. But other definitions can be used. For instance, an equipotential level of gravity (to a constant) perfectly defines the outer shape. Secondly, the Sun is a fluid body in rotation. It follows to first order an oblateness of the whole figure, and to other orders, deviations to sphericity. The diameter \( D \) under consideration must be thus identified. The semi-diameter \( (R, \text{radius, more frequently used}) \) is referred as equatorial, \( R_{eq} \), or polar, \( R_{pol} \), for which values are (Rozelot and Lefebvre, 2003): \( R_{eq} = 6.95991756 \times 10^8 \) m and \( R_{pol} = 6.95985861 \times 10^8 \) m (uniform rotation), or \( R_{eq} = 6.95991756 \times 10^8 \) m and \( R_{pol} = 6.95984386 \times 10^8 \) m (non-uniform rotation).

At last, on a pure physical point of view, as the distribution of matter is not uniform inside the Sun (from the core to the surface), as well as the distribution of the velocity rates, the outer shape shows distortions which are linked to the successive gravitational
moments. Hence, the solar radius, $R(t)$, must be a function of the latitude ($\theta$). As a consequence, all layers that constitute the Sun are not spherical (Fig. 1). This has been already recognized for instance for the tachocline (Gilman and Dikpati, 2002).

The knowledge of the value of the solar radius, once the definition is stated, is a key parameter not only in stellar physics but also in solar models. On very long-term evolution (several millennia), this has been recognized as the paradox of the faint young Sun. On shorter term (since around 1600 up to now), it has been shown that the solar radius may also evolves with time (see Fig. 2 in Pap et al., 2001, or Fig. 3 in Rozelot, 2001a; Rozelot, 2006, or Fig. 2 and 3 in Rozelot, 2001b, all upgraded from Toulmonde, 1997), likely on a very large periodic modulation, of about 110–120 years (from one extremum to the other one), the amplitude being not yet accurately determined. On shorter periods of time (ranging over some solar cycles), the temporal variability has remained unclear for a long time, but it has been shown recently that it is in antiphase with the solar cycle for layers lying at the very near surface of the Sun, and in phase for layers seated most deeper inside (Lefebvre and Kosovichev, 2005; Lefebvre et al., 2006a, 2007) (see Section 4.3).

The relevance of precise measurements of the Sun's shape can be summarized as follows:

- If $R(t)$ is known over a long period of time, ranging over several millennia, and even on undecennial cycles, then luminosity variations can be tackled (solar luminosity has increased over the life time of the Sun). By contrast, we do not know yet how radius variations on time scales ranging from seconds to hours, if they exist, may play a role in the luminosity variations. Hence, determination, in real time, of the so-called “asphericity-luminosity parameter”

$$w = \frac{\ln(R)}{\ln(L)}$$

is required. A table summarizing the estimated values of $w$ is given in Fazel et al. (2005) and Fazel (2007). The knowledge of this parameter is of high importance for the study of the Earth’s upper atmosphere.

- If $R(t)$ is known, then asphericities coefficients $c_n$ can be deduced, leading in principle to a determination of the solar gravitational moments $J_n$. The knowledge of these parameters is relevant to celestial mechanics and is required to set up precise ephemeris (due to the relation between $J_n$ and the inclination of the orbits of planets, i.e., spin–orbit couplings) in a General Relativistic description (Pireaux and Rozelot, 2003, 2005).

- If $R(t)$ and $R(t)$ are known, then the solar core dynamics can be inferred. This can be achieved either through ground–based observations where the Fried parameter is larger than 15–20 cm or through dedicated space missions, such as SDO (Solar Dynamics Observatory, expected launch by December 2009) or DynaMICCS/GOLF-NG (Dynamics and Magnetism from the Inner Core to the Corona of the Sun, expected to be launched by 2010–2012) (Turck-Chièze et al., 2005, 2009), and maybe by PICARD (expected launch, November 2009, and if accuracy is enough significant).

2. Observations of the solar shape

The solar shape is very difficult to observe, and hence very difficult to measure with the required astrometric accuracy. If Dicke and Goldenberg (1967) can be considered as a pioneer in this task, his first attempts at Princeton were not convincing. Several other measurements, made between 1974 and 1994 (see a review in Pireaux and Rozelot, 2003 or Rozelot and Rösch, 1996), lead to more reliable results. Up to the 1990s, only the oblateness was searched for. To summarize, it was shown that, if the Sun were rotating at a uniform velocity rate, the oblateness is

$$\Delta R = (R_0 - R) = 6 \text{ 187 m or 8.53 mas.} \quad (1)$$

The so-called $w$ parameter was first introduced by S. Sabatino in 1980 to assess a probable detection of climatically significant change of the solar constant; see “The Ancient Sun”, Pepin, Rohr., Eddy, J.A. and Merill, R.B., Pergamon Press (USA), 139–146.

Note that $\Delta R$ (Eq. (2)) is upper-bounded by 10.54 $\pm$ 0.25 mas as a maximum and 6.39 $\pm$ 1.31 mas as a minimum, according to the value adopted for the velocity rate (at the surface) (Rozelot, 2001a).
However, taking the differential rotation into account, the oblateness becomes
\[ \Delta R = (R_{eq} - R_{pol}) = 7370 \text{ m or } 10.15 \text{ mas}. \]  

(2)

It must be noted that the differential rotation increases the oblateness, in apparent contradiction with the theory of rotating stars (one would expect a lower flatness as the differential rotation decreases at the pole). This can be explained by a change in the radial velocity rate near 45° latitude (\(d\Omega/dr = 0\) at this latitude, with \(d\Omega/dr > 0\) at higher latitudes and \(d\Omega/dr < 0\) at lower latitudes).

Today, the best results concerning estimates of \(\Delta R\) are given through three different techniques. The results of the first one, deduced through balloon flights (in limited number) and the so-called “SDS” experiment, can be found in Sofia (2005). The second one, still into operation, has been developed at Mount Wilson Observatory (USA). Observations are based on a spectrographic analysis of the neutral iron line Fe I at 525 nm. Measurements have been recently re-analyzed by Lefebvre et al. (2004, 2006b).

The third one is developed at the Pic du Midi Observatory (F) by (2004, 2006b). Departures from a pure sphere are clearly seen: a depression, the polar shape remaining oblate. However, it can be seen an excess of asphericity found in the Mount Wilson data (120 mas around 20° of heliographic latitude with respect to 70° latitude); it can be interpreted by the spectral domain in which observations are made from the ground. A contribution of the upper layers of the photosphere can be suspected, as it was already found that the chromosphere maybe oblate (Auchere et al., 1998). This excess may also result from solar magnetic activity (Pecker, 1996), as do the frequency variations of the helioseismic modes, seen from limb observations made by RHESSI (Fivian et al., 2007, 2008). It must be noted that SDS experiments yield an oblateness in phase opposition with the solar cycle, in contradiction with other results: ground-based observations (heliometer) (Rozelot and Rösch, 1996), space observations (SOHO-MDI) (Emilio et al., 2007) (RHESSI) (Fivian et al., 2007).

Other results of the solar oblateness (mainly performed by Dicke and collaborators) can be found in Rösch et al. (1996) and are summarized in Table II given by Pireaux and Rozelot (2003). However, recent confrontation of the various available and reliable data lead to the conclusion that the solar oblateness varies in phase with solar activity (Rösch and Damiani, 2009), as first suspected through the heliometer measurements (Rösch et al., 1996), then confirmed via space observations (Emilio et al., 2007).

Dicke and Rösch can be thus considered as precursors in the field of the solar shape. The first one has undeniably set the basis of the underlying physics of the oblateness. Even if his papers were often examined critically, they triggered a great amount of ideas which have moved astrophysics forward. The second one carefully examined the conditions of solar diameter observations, such as blurring effects or displacement of the inflection point toward the inner part of the disk. He defined also the helioid as the whole outer solar shape, in an analogy with the Earth’s geoid.

Finally, a recent analysis of the data obtained at the Pic du Midi Observatory shows that the departures of the solar shape from a sphere reach about 20 mas. One of the first attempt to understand theoretically solar surface distortions was made by Lefebvre and Rozelot (2004) who showed that the thermal wind effect is one of the contributors at the solar surface. Note that the thermal wind (which is not the solar wind) is by the temperature difference between the pole and the equator and is equivalent to the Earth geostrophic effect, well studied by meteorologists.

3. How large are the temporal variations of the solar diameter?

On physical grounds (gravitational energy), temporal variability of the solar diameter cannot exceed some 10–15 mas peak to peak in amplitude. Callebaut et al. (2002) were certainly the firsts to point out that changes in solar gravitational energy, in the upper layers, necessarily involve limited variations in the size of

Fig. 2. Comparison between results deduced from the Mount Wilson data, over 30 years of analysis (left scale—upper curve) (Lefebvre et al., 2004; Lefebvre and Rozelot, 2004), and those of the Pic du Midi, obtained on September, 1st–4th, 2001, where exceptional conditions of seeing where encountered (right scale) (Rozelot et al., 2003). The observed solar limb contour does not follow an ellipsoidal shape, and shows deviations to sphericity, as theory states (Lefebvre et al., 2006b). See text for a possible explanation of the excess of amplitude at mid-latitudes.
the envelope. The mechanism can be described as follows, assuming hydrostatic equilibrium. Bearing in mind the definition of the gravitational energy \(E_g = -\frac{\Delta m}{r}G/rdm\), a thin shell of radius \(dr\) (or \(dm\)) in equilibrium under the gravitational force and the pressure gradient will expand or contract if any perturbation to these forces occurs. In Fazel et al. (2005, 2008), the authors improved the method and show that any variations of the size of the solar envelope must be less than some 11 km (\(\approx 15\) mas) of amplitude over a solar cycle, a value in agreement with those deduced from inversion of the helioseismic modes, or from space observations through the SOHO-MDI data analysis (Kuhn et al., 2004). It could be argued that the MDI instrument was not specifically designed for an astrometric purpose, and that the result obtained was reached after corrections for different effects (ageing, thermoelastic effects, etc.). This last reason gives weight to new space dedicated missions (see later).

Any other larger values is not consistent with astrophysical observations of other solar phenomena. For example, temporal irradiance changes observed at a level of \(\approx 1/\tau\) over the solar cycle could be explained by a \(\approx 200\) mas changes in the solar diameter, if this mechanism ought to play a unique role. Such a large value is not realistic, as it would automatically cancel all other physical explanations and among them, the magnetism of the surface, which is known to explain most (but not all) of the irradiance variations.

Li et al. (2006) developed a high-precision two-dimensional stellar evolution code for studying solar variability due to structural changes produced by varying internal magnetic fields of arbitrary configurations. They were able to show that the observed cyclic variations of solar irradiance, effective temperature, radius, and \(p\)-mode oscillation frequencies require a magnetic field component between 20 and 47 kG, peaked at \(r = 0.96R_S\), a stronger component of about 300 kG buried in the overshoot layer beneath the base of the convection zone cannot be ruled out. Thus the leptocline is maybe the key point to understand if large amplitude of the solar radius exist, they must be located at the extreme border of the limb. To this point, it is still difficult to disentangle between theories, and here again, space missions are needed.

As another example, consider for instance the multipolar gravitational moments of the Sun. The use of larger values of \(\Delta R(r)\) in models which are tested to other respects, would lead to major difficulties. Such is the case of the theory of lunar motion for which the inclusion of \(J_0\) estimates in a spin–orbital motion theory can be accurately confronted to observed lunar physical librations. As these librations are known with a precision of a few milliarcseconds, it results that \(\Delta R(r)\) is inevitably upper bounded by some \(10–15\) mas (Rozelot and Bois, 1998).

The next question the reader may ask is why solar astrolabes, distributed around the Earth (in France, Chile, Brazil and Turkey), are still measuring a diameter variability over the solar cycle of about 100–300 mas (sometimes more, up to 700 mas\(^7\)). A recent careful analysis, based on a statistical variographic analysis (Badache-Damiani and Rozelot, 2006; Damiani-Badache et al., 2007) showed that measurements made by astrolabes may be located around the Earth (in France, Chile, Brazil and Turkey), by some 10–15 mas (Rozelot and Bois, 1998).

The first quoted paper describes asphericities in the sub-surface layers: one asphericity is located around 0.78\(R_S\) (which is identified as the tachocline), and another one is located between 0.982 and 0.993\(R_S\), with two dipoles, at 0.986\(R_S\) and at 0.992\(R_S\); this last layer constitutes the leptocline (Fig. 2 in Godier and Rozelot, 2001). The second set of papers are based on the assumption that the temperature of the surface is nearly immutable, as suggested from observations made by Livingston at Kitt Peak (Livingston and Wallace, 2003; Livingston et al., 2005). It is shown that to model the remaining part of the irradiance variations (i.e. the part which is not coming from surface magnetism phenomena), it may exist a phase-shift in the \([dT, dR]\) plane, with a \([dT/\, dR]\) curve separating solar variations in antiphase (for temperature values below to 0.08 K), and in phase (for temperature values greater than 0.08 K) with solar irradiance variations. The third set of papers reports changes of the Sun's subsurface stratification inferred from helioseismic inversions (see Section 4.3), for which a clear phase change with depth is shown.

4. The solar shape and fundamental physics

Among all the fundamental parameters of the Sun (diameter, mass, temperature, etc.), the successive gravitational moments that determine the solar moments of inertia are still poorly known (Table 1). However, these moments have a physical meaning: they tell us how much the Sun's material contents deviate from a purely spherical distribution and how much the velocity rate differs from a uniform distribution. Thus, their precise determinations give indications on the solar internal structure.

The dynamic study of the gravitational moments until now was mainly based on solar observations (helioseismology, solar diameter) and solar models of rotation and density. Various methods (stellar structure equations coupled to a model of differential rotation, theory of the Figure of the Sun, helioseismology) lead to different estimates of \(J_0\), which agree as an order of magnitude but diverge as far as their precise values are concerned (Pireaux et al., 2005, Rozelot et al., 2004a, b).

4.1. Solar asphericities

The complexity of the rotation profile (Fig. 1, right) should reflect on the photospheric shape; in other words, the outer figure is highly sensitive to the interior structure and dynamics. Thus, in principle, accurate measurements of the limb shape distortions, which are called “asphericities” (i.e. departures from the “helioid”, the reference equilibrium surface of the Sun), combined with an accurate determination of the solar rotation provides useful constraints on the internal layers of the Sun (density, shear zones, surface circulation of the plasma, etc.). Alternatively, theoretical upper-bounds could be inferred for the flattening which may exclude incorrect/biased observations. Such asphericities can be computed according to the degree \(l\) and order \(n\) in the

\(^7\) The way astrolabe measurements are corrected from blurring, thermal or other effects remains unclear.
development of Laplace spherical harmonics in the general case of a rotating fluid body.

Let \( \rho \) be the density (function of the radius \( r \)) and denote with a subscript \( 0 \), the lowest order \( l \), spherically symmetric. Asphericities, described as

\[
c_i = - \frac{\rho_i}{d \rho^0/dr} \text{ (density)},
\]

\[
s_i = - \frac{p_i}{d p^0/dr} \text{ (pressure)}
\]

measure the perturbation (nonspherically symmetric) \cite{Armstrong and Kuhn, 1999} and are usually expressed in terms of the normalized potential defined by \( J_l = K \phi_{2l} \), where \( K = R_0^2 \) at the solar surface. The different gravitational moments can be written as

\[
J_{2l} = \frac{R_0}{GM_\odot} \phi_{2l}(R_\odot),
\]

where \( \phi_{2l} = 0 \) at the surface \( r = R_\odot \). The function \( \phi_{2l} \) is the solution of a differential equation requiring the knowledge of \( \rho(r) \) and \( \Omega(r, \theta) \), where \( \theta \) is the colatitude. A complete expression of \( \phi_{2l} \) and \( \phi_{4l} \) was provided by \cite{Armstrong and Kuhn, 1999} that uses the solar standard rotation law \( \omega_\odot = \omega_0 + \omega_2 \cos^2(\theta) \), which permits to deduce \( c_3 \):

\[
\begin{align*}
\frac{d^2 \phi_{2l}}{dr^2} + \frac{2}{r} \frac{d \phi_{2l}}{dr} - \frac{6}{r^2} \phi_{2l} &= 4 \frac{r^2}{M_\odot} \left\{ \phi_{4l} \frac{d \rho_0}{dr} \frac{2 \omega_0 \rho_0 \phi_{0} - 2 \omega_2 \phi_{0} \phi_{2} - r^2 \frac{d r^2}{dr} \left( \frac{2 \omega_2}{\rho_0^3} \phi_{0} \phi_{2} \right)}{3 \omega_0 \phi_{0} \phi_{2} \left( \frac{2 \omega_2}{\rho_0^3} \phi_{0} \phi_{2} \right)} \right\},
\end{align*}
\]

\[
\begin{align*}
\frac{d^2 \phi_{4l}}{dr^2} + \frac{2}{r} \frac{d \phi_{4l}}{dr} - 20 \frac{r^2}{r^2} \phi_{4l} &= 4 \frac{r^2}{M_\odot} \left\{ \phi_{4l} \frac{d \rho_0}{dr} \frac{2 \omega_0 \rho_0 \phi_{0} - 2 \omega_2 \phi_{0} \phi_{2} - r^2 \frac{d r^2}{dr} \left( \frac{2 \omega_2}{\rho_0^3} \phi_{0} \phi_{2} \right)}{3 \omega_0 \phi_{0} \phi_{2} \left( \frac{2 \omega_2}{\rho_0^3} \phi_{0} \phi_{2} \right)} \right\}.
\end{align*}
\]

4.2. Solar gravitational multipole moments

Observations allow to constrain analytical rotation models, in \( \theta \) and in depth. The first attempt to derive an analytical rotation law from helioseismic data has been made by \cite{Kosovichev, 1996}. Using such a law, several authors computed the gravitational moments (see Table 1), but discrepancies appeared, mainly for \( J_4 \). The discrepancy between the values obtained through different methods and authors can be explained by the use of different density models and by the way the differential equation (5) is integrated (through (6) and (7)). It can be seen that the method using helioseismic data leads to multipole moment values lower than those obtained by other methods. However, the hexadecapole moment, \( J_4 \), is much more sensitive than the quadrupole moment, \( J_2 \), to the distribution of angular velocity in the convective zone. \cite{Ajabshirizadeh et al., 2008} showed that the surface magnetism may reconcile the different approaches.

**Table 1**

<table>
<thead>
<tr>
<th>References</th>
<th>Method</th>
<th>( J_2 )</th>
<th>( J_4 )</th>
<th>( J_6 )</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ulrich and Hawkins (1981)</td>
<td>SSE + spots rotation law</td>
<td>( 10^{-15} ) \times 10^{-8}</td>
<td>( 0.2 ) \times 10^{-8}</td>
<td>( 4 \times 10^{-8} )</td>
<td>( J_8 = -4 \times 10^{-9} ), ( J_{10} = -2 \times 10^{-10} )</td>
</tr>
<tr>
<td>Gough (1982)</td>
<td>First determination of helioseismic rot. rates</td>
<td>( 36 \times 10^{-7} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Campbell and Moffat (1983)</td>
<td>Planetary orbits</td>
<td>( 5.5 \pm 1.3 ) \times 10^{-6}</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Landgraf (1992)</td>
<td>Astrometry of minor planets SDS experiment</td>
<td>( 0.6 \pm 5.8 ) \times 10^{-6}</td>
<td>( 9.8 \times 10^{-7} )</td>
<td>( 4 \times 10^{-8} )</td>
<td></td>
</tr>
<tr>
<td>Lydon and Sofia (1996)</td>
<td></td>
<td>( 1.84 \times 10^{-7} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paternò et al. (1996)</td>
<td>SSE + empirical rotation law and SDS</td>
<td>( 2.22 \times 10^{-7} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pijpers (1998)</td>
<td>SSE + GONG and SOI/MIDI data</td>
<td>( 2.14 \times 0.09 ) \times 10^{-7}</td>
<td>( 2.23 \times 0.09 ) \times 10^{-7}</td>
<td>( 2.14 \times 0.06 ) \times 10^{-7}</td>
<td></td>
</tr>
<tr>
<td>Armstrong and Kuhn (1999)</td>
<td>Vect. spher. harm. numerical error</td>
<td>( -0.222 \times 10^{-6} ), 0.002 \times 10^{-6}</td>
<td>( 3.84 \times 10^{-9} ), 0.4 \times 10^{-9}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Godier and Rozelot (1999)</td>
<td>SSE + Kosovichev law</td>
<td>( 1.6 \times 10^{-7} ) Note 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Roxburgh (2001)</td>
<td>SSE + 2 models of rotation law</td>
<td>( 2.208 \times 10^{-7} ), ( 2.206 \times 10^{-7} )</td>
<td>( -4.46 \times 10^{-9} ), ( -4.44 \times 10^{-9} )</td>
<td>( -2.80 \times 10^{-10} ), ( -2.79 \times 10^{-10} )</td>
<td>( J_8 = 1.49 \times 10^{-11} ), ( J_9 = 1.48 \times 10^{-11} )</td>
</tr>
<tr>
<td>Rozelot (2001a)</td>
<td>Theory of Figures</td>
<td>( -6.13 \pm 2.52 \times 10^{-7} ) Note 3</td>
<td>( 3.4 \times 10^{-7} ) Note 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rozelot and Lefebvre (2003)</td>
<td>Theory of Figures</td>
<td>( -6.52 \times 10^{-7} )</td>
<td>( 4.20 \times 10^{-7} )</td>
<td>( -9.46 \times 10^{-9} )</td>
<td>( J_6 = 2.94 \times 10^{-11} )</td>
</tr>
<tr>
<td>Rozelot et al. (2004)</td>
<td>SSE + subsurface gradient of rotation (SGR)</td>
<td>( -2.28 \times 10^{-7} \pm 15% )</td>
<td>( 4.22 \times 10^{-7} \pm 20% ) very sensitive to SGR</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The order of magnitude of \( J_2 \) is \( 2.4 \times 10^{-7} \); \( J_4 \) is very sensitive to the subsurface gradient of rotation: an estimate of \( (4-7) \times 10^{-7} \) seems better match the observations.

Note 1: SDS stands for solar disk sextant, SSE stands for stellar structure equations.

Note 2: Density model dependent.

Note 3: The apparent large error comes from the fact that the value is a weighted average of several rotation rates.

Note 4: A mistake has been made in \cite{Godier and Rozelot, 2001}; the second term of \( J_4 \) (i.e. \( m A_4 \)) was incorrectly multiplied by "\( f \)" in the computations.
between the theory of Figures and numerical integration of Eq. (5): it seems that \( J_4 \) obtained by the first theory matches observations better. If we adopt \( (2.4 \pm 0.4) \times 10^{-7} \) as a good estimate for \( J_2 \), by consequence \( J_4 \) results to be very sensitive to the subsurface gradient of rotation: an estimate of \( (4-7) \times 10^{-7} \) seems to be in good agreement with the observations. Kosovichev (1996) noted that a subsurface shear layer appears a result of matching the obtained helioseismically internal rotation with the surface rotation. Hence, we suspected that the shallow layer near the surface may play an important role (Fig. 6).

4.3. The leptocline

By analyzing the temporal variation of \( f \)-mode frequencies for 1996–2004, Lefebvre and Kosovichev (2005) have shown changes in the Sun's subsurface stratification. They have found a variability of the “helioseismic” radius in antiphase with the solar activity, the strongest variations of the stratification being just below the surface, around \( 0.995 R_{\odot} \). On another hand, the radius of the deeper layers of the Sun, between \( 0.975 \) and \( 0.99 R_{\odot} \), change in phase with the 11-yr cycle (Fig. 5). A more careful analysis of these
f-modes shows variations in the even-$a$ coefficients, of non-negligible amplitude, both with the frequency and the cycle, that imply the existence of asphericities in the subsurface layers (Bedding et al., 2007; Lefebvre et al., 2006c). The conclusion is that this interface layer corresponding to the border between the interior of the Sun and its atmosphere is the seat of important physical phenomena (in addition to non-homologous radius changes in time and in depth), such as: shearing, disturbance of the turbulent pressure, constraints upon the magnetic field, processes of ionization and, likely, inversion of the radial gradient of rotation and some tiny variations of the luminosity (Fig. 6).

Even if this layer could be more complex, involving another shell of some oblateness at the very near surface (presently undetectable with f-modes), located at around $0.99\,R_{\odot}$, the proof is now made that the leptocline is a new and crucial zone that cannot be avoided in investigating the global properties of the Sun and its evolution on time scales of the order of months or years.

4.4. Solar radius and gravitational energy variations

As previously mentioned, the gravitational energy can be computed in the case of a non-spherical Sun that leads to a
variation of the solar luminosity $L$ with $dR$ according to a development of order $n$ (Fazel, 2007). This author made two computations, one with $n = 1$ (monotonic expansion with radius) and the other one with $n = 2$ (non-monotonic expansion). Results are given in Table 2 for two values of $\Delta L/L$: the usual adopted value, 0.0011, using TSI composite data from 1987 to 2001 (Dewitte et al., 2005); mean value of solar constant $C_0 = 1366.495$ W/m$^2$; and 0.00073, determined through a re-analysis of the composite TSI data over the period of time 1978–2004 (Frölich, 2005); mean value $C_0 = 1365.993$ W/m$^2$. For $n = 2$ (the most likely case consistent with recent other results), the estimate of $\Delta R$ is smaller than the 8.9 km obtained in the case of a spherical Sun by Callebaut et al. (2002). However, the $\Delta R/R$ agrees with that of Antia (2003), i.e. $\Delta R/R = 3 \times 10^{-6}$, who used $f$-mode frequencies data sets from MDI (from May 1996 to August 2002) to estimate the solar seismic radius with an accuracy of about 0.6 km (see also among other authors Antia, 1998; Schou et al., 1997, for such a determination of the solar seismic radius to a high accuracy).

Two points emerge from the analysis of the data. The first concerns the “helioseismic radius” which does not coincide with the photospheric one, the photospheric estimate being always larger by about 300 km (Brown and Christensen-Dalsgaard, 1998). This point would require more specific attention in the future (see also footnote 1).

The second issue addresses the shrinking of the Sun with magnetic activity as pointed out by Dziembowski et al. (2001), using $f$-mode data from the MDI instrument on board SOHO, from May 1996 to June 2000. They found a contraction of the Sun’s outer layers during the rising phase of the solar cycle and inferred a total shrinkage of no more than 18 km. Using a larger data base of 8 years and the same technique, Antia and Basu (2004) set an upper limit of about 1 km on possible radius variations (using data sets from MDI, covering the period of May 1996 to March 2004). However, they demonstrated that the use of $f$-modes frequencies for $l < 120$ seems to be unreliable.

As a consequence of the above discussion it appears that the luminosity changes are likely produced in the outer shallow layer of the Sun, and that the leptocline might be the seat of the observed $1/\alpha$ variations of the irradiance.

### 4.5. Results from ground-based observations and space missions

Solar asphericities mainly described by the first two coefficients $c_2$ and $c_4$ can be observed. An estimate of these two coefficients at the Sun’s surface derived from SOHO-MDI space-based observations are at the surface (Armstrong and Kuhn, 1999):

$$c_2 = (-5.27 \pm 0.38) \times 10^{-6} \quad \text{and} \quad c_4 = (1.3 \pm 0.51) \times 10^{-6} \quad \text{(8)}$$

These results were obtained by measuring small displacements of the solar-limb darkening function (details are given in Kuhn et al., 1998), and the $c_0$ coefficients are comparable to an isodensity surface level (see Eq. (3)). From Earth-based observations obtained by means of the scanning heliometer installed at the Pic du Midi Observatory (F), we also obtained estimates of $c_2$ and $c_4$ coefficients. The mean ponderated values, computed over three years, are

$$c_2 = (-6.56 \pm 0.18) \times 10^{-6} \quad \text{and} \quad c_4 = (+2.7 \pm 0.6) \times 10^{-6} \quad \text{(9)}$$

which indicates (Section 2) a slight bulge extending from the equator to the edge of the royal zones (about 40° of latitude), with a depression beyond (at the poles, the ellipsoidal figure prevails).

Such a distorted shape, not exceeding some 20 mas of amplitude, can be interpreted through the combination of the quadrupole and octopole terms, which, as shown previously, directly reflect the non-uniform velocity rate at the surface (and depth). Moreover, this distribution implies a thermal wind effect, from the poles toward the equator (Lefebvre and Rozelot, 2004). The observed value of $c_2$, $-6.6 \times 10^{-6}$, is not too far from the theoretical one, $\approx -2/(3f) \approx -5.9 \times 10^{-6}$ with $f = 8.9 \times 10^{-6}$ based on a solar model constrained by a differential rotation law. It agrees also with the SOHO-MDI observations (see Eq. (8)) and the theoretical estimate deduced from a vector harmonics solution (Armstrong and Kuhn, 1999), $-5.87 \times 10^{-6}$. It remains difficult to match the coefficient $c_4$ with the theory, which predicts $+12(35f^2$ for a uniform rotation law, and 0.616 $\times 10^{-6}$ for a differential one. The most likely explanation is that the shape coefficient $c_4$ is very sensitive to surface phenomena and differential rotation (as for $l_1$).

Only space dedicated satellites will be able to definitively provide an answer to these questions.

### 4.6. Temporal variations of the solar shape

If we are beginning to understand the significant physical character of the leptocline, we are far to know if the asphericities are variable with time. It is difficult to reasonably think that a temporal variability of such parameters might be due to the temporal variation of the internal structure. Lefebvre et al. (2006a, 2009) and Bedding et al. (2007) reported first analysis concerning the temporal dependence of even-\(\alpha\) coefficients. The available data (SOHO $f$-modes) permits to have access to the first 18 even-\(\alpha\) coefficients, but for sake of clarity, only the first three ones ($\delta_3 \alpha$) were computed (Fig. 7).

Each curve is an average difference over a specific year computed by reference to the minimum year of activity, 1996 (the errorbars are not shown). The first graph on the left ($\delta_3 \alpha$) shows a negative trend, a frequency behavior almost flat and a clear behavior with the cycle in antiphase. The second graph dedicated to the variation ($\delta_3 \gamma$) of $\gamma$ shows a slight increase with the frequency, a change of sign at high frequency and no clear variations with the cycle. As far as the variation ($\delta_3 \gamma$) of $\gamma$ is concerned, the dispersion is too big to say anything.

In such a way, the outer shape would be time dependent, and this could explain also some tiny fluctuations of the irradiance. It is thus of interest to explore the whole chain, starting from the core up to the surface, to well understand the mechanisms of solar

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8 A first explanation (based on a temporal average) of such a result lies in the proposal made by Peckar (1996): “in the royal zones the existence of spots diminishes the solar brightness and thus the measured radius; but this effect is compensated and reinforced by the existence of faculae, which extends higher in latitudes. The measured radius is globally greater at the latitudes where faculae are statistically more numerous. At highest latitudes, no spots and faculae appear any longer, and the measured radius is consequently reduced.”
activity, then to get a better prediction, and then to understand how the solar output may influence the atmosphere of our planet. One can judge such an investigation as ambitious, but we are today compelled to carefully examine all the sources of the solar variability to get a scientific opinion on the solar forcing—and even if it is to reject one of the processes.

4.7. Solar shape and general relativity

If, from a physical point of view, the multipolar moments lead to distortions of the solar surface (asphericities), they have also a dynamic role in the light deflection or in celestial mechanics. In the ephemerides computation, the determination of \( J_2 \) is strongly correlated with the determination of the post-Newtonian parameters (PPN) characterizing the relativistic theories of the gravitation. Lastly, the ignorance of \( J_2 \) is also a barrier to the determination of models of evolution of the solar system on the long term.

The relativistic aspects are crucial in the dynamic approach of the solar parameters and open interesting prospects for the future. In this context, it is interesting to obtain a dynamic constraint of \( J_2 \), independent of the solar models of rotation and density, being used thereafter to force the solar models. Such a study is relevant in the scope of space missions such as BepiColombo (better determination of the PPN; possible measurement of the precession of the apside line of Mercury as a function of \( J_2 \)), GAIA (better determination of the PPN, possible decorrelation PPN-\( J_2 \) thanks to the relativistic advance of the perihelion of planets and minor planets) and obviously GOLF-NG (precise determination of the rotation of the core where the quasi-totality of the mass is concentrated). Another key parameter of the solar models, which could also be constrained in a dynamic way is its spin, from the spin–orbit couplings which is introduced in celestial mechanics. From present solar system experiments (Lunar Laser Ranging, Cassini Doppler experiments, etc., see Pireaux and Rozelot, 2003), it turns out that General Relativity is not excluded by those, as shown in the most up to date values in Fig. 8a. However, General Relativity would be incompatible with the Mercury perihelion advance test if \( J_2 = 0 \) was assumed. But with a non-zero \( J_2 \), General Relativity agrees with this latter test, and there is still room for an alternative theory too (see Fig. 8b). Space missions such as BepiColombo or GAIA should provide the necessary \( J_n \) measurements.

With regard to the solar core dynamics, the subject is of high priority for new investigations. Here again, space-dedicated missions, such as DynaMICCS/GOLF-NG in a joint effort with SDO (Solar Dynamics Observatory), scheduled in a next future, should provide a new insight on the question.

5. Conclusion

From an historical point of view, the question of whether the diameter of the Sun evolves with time or not is very fertile. On time scales of the order of the millennium, the question of the solar luminosity can be tackled. The time interval from the medieval era up to now, the debate is not really closed. On the basis of Tobias measurements, Wittmann et al. (1981) claimed that no secular solar diameter decrease can be inferred. We are more in favor of a long-term modulation, the Sun’ diameter being bigger during periods of lower activity, such as during the Maunder Minimum, and smaller in periods of more intense activity such as at the present period. On smaller time scales (over few activity cycles), models are still needed, but new input will come, paradoxically, from a better knowledge of the temporal evolution of the limb fluctuations (including solar diameter variations). On time scales of the order of months (or years), the variability is upper bounded by some 10 mas (to 15 mas as an upper bound). Such estimates, deduced on physical grounds (mainly from gravitational energy variations) cannot be reconciled with solar astrolabe measurements, which give a variability of one, even two orders or magnitude greater. Thus, it seems that the model proposed by Sofia et al. (2005), that shows an increase of the solar radius by a factor of approximately 1000 from a depth of 5 Mm to the solar surface, is not consistent, in spite of its achieved formalism that includes magnetic field.\(^{10}\) The question of the phase depends on the depth, the diameter variations being in

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\(^{10}\) Recent papers on large amplitude solar radius show that the filtering process maybe evoked (Djafer et al., 2008).
antiphase at the surface and progressively setting in phase below 0.99Rₚ.

The study of asphericities, directly linked with solar gravitational moments, is not only crucial for solar physics but also for astrometry (when computing light deflection in the vicinity of the Sun), celestial mechanics (relativistic precession of planets, planetary orbit inclination and spin–orbit couplings) and for future tests of alternative theories of gravitation (correlation of J₂ with post-Newtonian parameters; Pireaux and Rozelot, 2003, 2005).

Another issue is the solar changing shape. It has been shown that the outer solar shape differs significantly from a sphere, with a bulge at the equator, and a depressed zone at higher latitudes (the change being around 45°, due to the reversal of the radial velocity rate); the whole shape remains oblate at the pole. The Pic du Midi observations show a variability of the whole oblate shape in phase with solar activity (Rozelot and Rösch, 1996) that is compatible with the above mentioned long-term solar diameter modulation. Accurate measurements from space observations are needed. They can be achieved by next generation of satellites, such as PICARD, for which one of the author was one of the first initiators of this program (Damé et al., 2001; Lefebvre and Rozelot, 2001), DynaMICCS (Turck-Chièze et al., 2005, 2009), SDO, or even balloon flights (Sofia, 2005) (new flights are scheduled by 2010). On a longer term, GAIA, a space mission expected to flight by the end of 2012, will allow to estimate the perihelion precessions of Mercury, Icarus, Talos and Phaeton. In this case, it will be possible to separate the relativistic and the solar contributions in the perihelion advance, so that gravitational moments could directly be determined from dynamics, without the need of a solar model. Note also that presently, dynamical estimates of J₂ are strongly correlated with the estimate of the post-Newtonian parameter β, which together with other PN parameters, characterizes relativistic theories of gravitation in observational tests. However, future PPN testing space missions, as well as non-dedicated missions like GAIA, might help solve this problem.

According to the temporal variation of the f-mode frequencies, the region underlying the surface is stratified in a thin double layer, interfacing the convective zone and the surface. This “leptocline” is the seat of many phenomena: an oscillation phase of the seismic radius, together with a non-monotonic expansion of this radius with depth, a change in the turbulent pressure, likely an inversion in the radial gradient of the rotation velocity rate at about 45° in latitude, opacities changes, super-adiabatic stratification, the cradle of hydrogen and helium ionization processes and probably the seat of in situ magnetic fields (Lefebvre et al., 2006a). Fig. 6 shows a schematic view of the complex physics in this shear zone. Finally, we pose a question. Why helioseismology always lead to smaller values of the parameters under investigations? The rotation velocity rate at the surface is smaller than the rates obtained through other techniques (at the surface, see Table given in Lefebvre and Rozelot, 2004), as well as the quadrupole moment estimates and the radius itself. To disentangle all these points, we need to wait space results. First, from the SDO (Solar Dynamics Observatory) mission, with precise resolved velocity oscillation measures (HMI/SDO instrument). Second, from the PICARD satellite (as seen before). Then, from GOLF-NG, the GOLF/SOHO successor, as proposed in a future mission like DynaMICCS, for which the final aim is to reveal the complete 3D vision of the Sun. And if accepted, a Brazilian mission called SPHERIS, specifically designed for astrometric measurements of the solar radius, at a 2020 horizon. To be complete, let us mentioned the ASTROMETRIA project, from the Pulkovo Observatory (Abdussamatov, 2009).

The problem of determining the temporal diameter evolution of the Sun is still rich and fascinating. We hope that the present argument could be of interest for a broader community for a deeper investigation in this research field.

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