Is the Lee constant a cosmological constant?

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It is shown that the gauge theories of elementary particles with spontaneous symmetry breaking introduce into the gravitation equations a cosmological term that varies with time. The change in this term during the evolution of the universe amounts to at least 49 orders of magnitude.

(1)

\[ R_{ik} - \frac{1}{2} \delta_{ik} R = \kappa T_{ik} + \delta_{ik} \Lambda \]

contains in the general case a term with the cosmological constant \( \Lambda \). This constant is regarded as universal and independent of the state of the universe. \(^{(1,2)}\)

The purpose of the present paper is to show that an inevitable consequence of the ideas of the presently developing unified theory of weak and electromagnetic interactions is a dependence of \( \Lambda \) on the temperature of the medium. For the evolving hot universe this means a dependence of \( \Lambda \) on the time.

1. If, as usual, \( T_{ik} \) is taken to mean the energy-momentum tensor of matter and radiation, then the quantity \( g_{\mu\nu} \Lambda / \kappa \) can be treated as an energy-momentum tensor of vacuum. \(^{(3)}\) In elementary-particle theory, however, the energy of the vacuum and its pressure (equal to its energy density taken with a minus sign) are determined only accurate to an arbitrary constant. Therefore the "old" theories of elementary particles have yielded no information whatever on the value of \( \Lambda \).

The situation has changed radically after the appearance of gauge theories with spontaneous symmetry breaking, on the basis of which one can hope, with a great degree of justification, to construct the future unified theory of weak, electromagnetic, and possibly strong interactions. \(^{(3)}\) By its very meaning, spontaneous symmetry breaking is the result of the instability of the "usual" vacuum, which becomes restructured into a new state that is energywise more favorable. The corresponding change in the energy of the vacuum depends on the temperature of the medium, decreases with increase of this energy, and vanishes at a certain critical temperature \( T_c \). \(^{(4)}\) By the same token, the energy of the vacuum becomes temperature-dependent, and so does consequently the cosmological constant \( \Lambda \). Although this constant is defined, as before, only accurate to an arbitrary constant term, the difference \( \Lambda(T_c) - \Lambda(0) \) now has a perfectly defined value. It is precisely this difference which will be discussed below.

2. To calculate the character of the dependence of \( \Lambda \) on the temperature \( T \), we consider a model that is contained (with small variations) as a component part in all the models of the unified theory, and ensures the spontaneous symmetry breaking. The Lagrangian of this model is

\[ L = \partial_{\mu} \phi \partial_{\mu} \phi + \mu^2 \phi^4 + \frac{\lambda}{4} \phi^4 \]

where \( \phi \) is a complex scalar field. Owing to the "incorrect" sign of the term \( \mu^2 \phi^4 \), the usual solution yields at \( T = 0 \) particles that have an imaginary mass \( m = i \mu \), i.e., tachyons. This solution is unstable; to obtain a stable solution we must make the substitution

\[ \phi = (1/\sqrt{2})(\phi_1 + i \phi_2 + \sigma) \]

after which \( L \) goes over into

\[ \tilde{L}_c = \frac{1}{4} \mu^2 \phi^2 - \frac{\lambda}{4} \phi^4 \]

where \( \tilde{L}_c \) is the operator part of \( L_c \), and \( \mu^2 \phi^2 / (\lambda \phi^4 / 4) \) is a numerical term corresponding to the change in the energy density of the vacuum (with the sign reversed). The symmetry breaking parameter \( \sigma = \sigma(T) \) depends on the temperature, with \( \sigma(0) = \mu / \sqrt{\lambda} \) and when \( T \) increases the quantity \( \sigma(T) \) decreases, vanishing throughout at \( T > T_c \), where \( T_c \sim \sigma(0) \). \(^{(1,4)}\) We denote the energy density of the vacuum at the temperature \( T \) by \( \epsilon(T) \). It follows then from (3) that

\[ \epsilon(T) = \epsilon(0) + \frac{\lambda}{4} (\sigma(0)^2 - \sigma(T)^2) \]

whence

\[ \Lambda(T) = \Lambda(0) + \frac{\lambda}{4} (\sigma(0)^2 - \sigma(T)^2) \]

3. For a numerical estimate we assume \( \sigma(0) \) the same value as in Weinberg's model, \(^{(1,6)}\) \( \sigma(0) \sqrt{2} G = 250 \) GeV (here \( G \) is the Fermi weak-interaction constant). For the quantity \( \lambda \) we have an experimental bound \( \lambda < 10^{16} \). Then at \( T > T_c \), the value of \( \epsilon(T) \) is larger than \( \epsilon(0) \), by an amount \( \approx 10^{45} \) erg/cm\(^3\) or, in grams, by \( \approx 10^{21} \) g/cm\(^3\). At the present time, the absolute value of \( \epsilon \) does not exceed \( 10^{28} \) g/cm\(^3\). Thus, at a sufficiently high temperature we have \( \Lambda(T) > \Lambda(0) \), i.e., we can state the value of \( \Lambda(T) \), even though we do not know the value of \( \Lambda(0) \), viz., \( \Lambda(T) > \Lambda(T_c) \). In particular, it follows from the obtained estimates that \( \Lambda(T) \sigma(0^2) > 10^{-6} \) cm\(^2\), whereas at present \( |\Lambda| \leq 10^{55} \) cm\(^2\). This means that during the time of the evolution of the universe the cosmological constant has changed by more than 49 orders.

To be sure, almost the entire change occurs near \( T_c \). \(^{(4)}\) In this region, the vacuum energy density is lower than the energy density of matter and radiation, and therefore the temperature dependence of \( \Lambda \) does not exert a decisive influence on the initial stage of the evolution of the universe. At the same time, the fact that \( \Lambda \) definitely differs from zero at a definite period of the existence of the universe makes speculations concerning a nonzero value of \( \Lambda \) in the present epoch more likely. In any case, it follows from the foregoing that, by discarding from the very outset the cosmological term in Einstein's equation, as was the custom even a few years ago (see also \(^{(7)}\)), we would
Radiative corrections to leptonic decays of hadrons in renormalizable models of weak interactions

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It is shown that radiative corrections to the ratio of the vector constants of $\beta$ and $\mu$ decays in renormalizable theories turn out to be essentially the same as in the usual scheme with the intermediate vector boson. Radiative corrections to $\pi_{\tau\nu}$ decays are also considered.

Since the radiative corrections turn out to be finite in renormalizable theories of weak interactions, this raises the question whether a choice among different schemes can be made by investigations the contributions made by higher approximations to the amplitudes of leptonic decays of hadrons.

We begin with the question with the radiative corrections to the ratio $G^\mu_\gamma/G^\nu_\gamma$ of the vector constants of $\beta$ and $\mu$ decays. As is well known,\(^{(1)}\) in local four-fermion theory the electromagnetic correction $G^\mu_\gamma$ diverges logarithmically. Introduction of the intermediate vector boson makes this quantity finite even within the framework of the ordinary renormalizable theory. The cut-off parameter is then replaced by the $W$-boson mass.\(^{(2)}\) It can be shown that the result does not depend on the magnetic moment of the $W$ boson. The final expression takes the form (if we disregard the contribution of the soft quanta, allowance for which does not depend on the model)

$$\frac{G^\mu_\gamma}{G^\nu_\gamma \cos \theta} = 1 - \frac{3a}{8\pi} (1 + 2\overline{Q}) \ln \frac{\mu^2}{m^2}.$$  \hspace{1cm} (1)

Here $\theta$ is the Cabibbo angle, $\overline{Q}$ is the average charge of the isodoublet of the elementary fields that enter in the weak current, $m_\pi$ is the characteristic hadron mass. In the derivation it is assumed that the mass of the $W$ boson is much larger than all the hadron masses, $m_\mu > m_\pi$, and that at a virtual $\gamma$-quantum momentum $q \gg m_\pi$ the behavior of the amplitude $T_{\mu\nu}$ of the process $\gamma p \rightarrow W^+ n$ is determined by the commutator of the electromagnetic and weak currents. In other words, it is assumed that in this momentum region the strong interactions are inessential. Expression (1) contains an uncertainty $\sim \alpha/\pi$, since a determination of the exact value of $m_\pi$ calls for detailed information on the amplitude $T_{\mu\nu}$ in the non-asymptotic region.

It is clear that in renormalizable theories the contribution of the virtual $\gamma$-quantum to the ratio $G^\mu_\gamma/G^\nu_\gamma$ is also described by formula (1). In these theories it is necessary to account for, besides the electromagnetic field, also the contributions of the neutral vector field $Z$ (in certain schemes) and of the scalar field $\sigma$, and also the corrections to the vertices $W_{\mu\nu}$ and $W_{\sigma}$ due to the $W$ boson.

However, none of the additional corrections contain a large logarithm if one makes the natural assumption concerning the masses $\mu_\gamma \sim \mu_\mu \sim m_\mu$. Indeed, the logarithm $\ln (\mu_\mu/m_\sigma)$ in expression (1) stems from the region of virtual momenta from $m_\pi$ to $m_\mu$. On the other hand, if the $\gamma$ quantum is replaced by a particle with mass $\mu > m_\pi$, then the essential region is from $\mu$ to $\mu_\mu$. Therefore all the new corrections do not exceed the uncertainty $\sim \alpha/\pi$ contained in formula (1).

More accurately speaking, the radiative corrections due to the $\sigma$ field turn out to be much smaller, $\sim (\alpha/\pi) \times (m_\pi/m_\sigma)^2$, since the constants for the interaction between these fields and hadrons or leptons contain the ratios of the particle masses to $m_\pi$.

As to the contributions of the $W$ and $Z$ bosons to the radiative corrections, they can be obtained by the same method\(^{(1)}\) as the electromagnetic contribution. The correction to the vertices connected with the $W$ boson turn out to be the same for the $\beta$ and $\mu$ decays. Allowance for the $Z$ boson in the Weinberg model\(^{(2-4)}\) leads to the increment

$$\frac{3a}{8\pi} (1 + 2\overline{Q}) \ln \frac{\mu^2}{\mu_\nu^2}$$

in the right hand side of (1). Thus, allowance for the $Z$
Erratum: Is the Lee constant a cosmological constant?  
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The title of the article has been mistranslated. The correct title is “Is the Cosmological Constant a Constant?” The translation editor regrets this mistake.