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THE LUMINOSITY FUNCTION AND STELLAR EVOLUTION

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ABSTRACT

The evolutionary significance of the observed luminosity function for main-sequence stars in the solar neighborhood is discussed. The hypothesis is made that stars move off the main sequence after burning about 10 per cent of their hydrogen mass and that stars have been created at a uniform rate in the solar neighborhood for the last five billion years.

Using this hypothesis and the observed luminosity function, the rate of star creation as a function of stellar mass is calculated. The total number and mass of stars which have moved off the main sequence is found to be comparable with the total number of white dwarfs and with the total mass of all fainter main-sequence stars, respectively.

I. INTRODUCTION

Since Baade's early work on the two types of stellar populations, much has been learned about stellar evolution from studies of population II systems. Of particular importance are the H-R diagrams of globular clusters (Arp, Baum, and Sandage 1952), where main-sequence stars brighter than M_v about 3.5 are missing almost completely and a subgiant branch, leading to a giant branch, starts at the same magnitude. These results were explained theoretically by Sandage and Schwarzschild (1952), on the assumption of very poor mixing between the central core and the outer layer of a star. According to their theory, the more luminous main-sequence stars, which have converted more than about 12 per cent of their mass (the Schönberg-Chandrasekhar limit) from hydrogen into helium in their central core, change their internal structure and rapidly move off the main sequence toward the red-giant region. The amount of energy released in the conversion of 1 gm of hydrogen into helium is known from nuclear data to be 6.2×10^{18} ergs. For a star with $M_v = 3.5$ and mass about $1.3\ \mathfrak{M}_\odot$ it takes about 6×10^9 years for the nuclear conversion of 12 per cent of the stellar mass. The absence of more luminous main-sequence stars in a globular cluster then indicates that the cluster is about 5×10^9 years old, which agrees well with other estimates of the age of our galaxy. Further, the fact that the main sequence peters out fairly suddenly indicates that all the stars of a cluster have practically the same age. Because of this uniformity in age and because of the rapid evolution of stars along the subgiant branch, globular clusters mainly give information on the evolution of stars in a narrow range of main-sequence luminosity and hence a narrow range of mass (near $1.3\ \mathfrak{M}_\odot$).

Population I systems, on the other hand, the solar neighborhood in particular, contain stars with a large range of ages. From a large amount of observational data on stars in our neighborhood, tolerably accurate values are available for the luminosity function $\phi(M_v, \text{Sp})$, the density of stars of absolute visual magnitude between $(M_v - \frac{1}{2})$ and $(M_v + \frac{1}{2})$ in a certain range of spectral class. This luminosity function depends on three factors: (i) $\xi(\mathfrak{M})$, the relative probability for the creation of stars of mass near $\mathfrak{M}$ at a particular time; (ii) the rate of creation of stars as a function of time since the formation of our galaxy; and (iii) the evolution of stars of different masses after they have burned out an appreciable fraction of their hydrogen mass and have left the main sequence.

One cannot, of course, derive these three factors from a knowledge of the luminosity function. But we can test whether the following series of hypotheses is compatible with the luminosity function: (*a*) The rate of formation of stars in the solar neighborhood has

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been uniform since the beginning of our galaxy (in contrast to population II systems, no very large number of stars were formed simultaneously in the beginning). (*b*) The factor $\xi(\mathfrak{M})$ is a smoothly varying function of mass $\mathfrak{M}$, independent of time. (*c*) Stars do not change their mass appreciably during their evolution, except at the last stages (accretion of matter, loss of matter by corpuscular radiation, etc., are unimportant). (*d*) Most stars are poorly mixed and move off the main sequence toward cooler surface temperature as soon as about 12 per cent of their mass has changed from hydrogen into helium.

II. OBSERVATIONAL DATA

The total luminosity function $\phi_t(M_v)$ is defined by

$$dN = \phi_t(M_v)\, dM \,, \tag{1}$$

where dN is the number of stars of all spectral classes per cubic parsec of absolute visual magnitude between M_v and $(M_v + dM)$. Among others, Van Rhijn (1925, 1936) has determined $\phi_t(M_v)$ between $M_v = -6$ and $M_v = +13$ from extensive observational data. For the low-luminosity part of this range, Luyten (1939, 1941) more recently has derived $\phi_t(M_v)$ from a detailed survey of faint stars in the Southern Hemisphere. We adopt a compromise between Van Rhijn's and Luyten's luminosity functions, similar to that of Bok (1941). This luminosity function, $\phi(M_v)$, is given in Table 2.

TABLE 1

FRACTION f OF MAIN-SEQUENCE STARS (TYPE EARLIER THAN Sp_d)

	M_v								
	−4.5	−3.5	−2.5	−1.5	−0.5	+0.5	+1.5	+2.5	+3.5
Sp_d..........	B0	B3	B6	B9	A1	A6	F0	F8	G7
f............	0.10	0.25	0.48	0.51	0.43	0.40	0.60	0.70	0.90

We shall only consider the luminosity function in the range $M_v = -4.5$ to $+13.5$. Luyten's work indicates that the luminosity function decreases for M_v greater than 13 or 14, although this does not necessarily mean that the total number of these faint stars is small. Our adopted luminosity function does not refer to stars at a homogeneous distance from the galactic plane. Work by McCuskey (1951) and others indicates that the luminosity functions for various distances from the galactic plane differ only in minor details from Van Rhijn's up to distances of about 500 parsecs, except possibly for $M_v < -2$.

We shall not consider the detailed luminosity–spectral type function $\phi(M, \mathrm{Sp})$ (cf. Luyten 1936), but we shall want the luminosity function for main-sequence stars alone, separated from the giants. Trumpler and Weaver (1953) give the relative frequency of stars of different spectral types with the same value of M_v, based on results of Strömberg and of Öpik for stars brighter than apparent visual magnitude +6. The resolving power of this data is not sufficiently high to give a clear separation between main sequence and subgiants. For each value of M_v we chose, somewhat arbitrarily, a dividing spectral type Sp_d, shown in Table 1. Comparison with a table (Keenan and Morgan 1951) of average values of M_v on the two-dimensional spectral type–luminosity class classification shows that stars of type earlier than Sp_d should contain most of the main-sequence stars (class V), a reasonable fraction of the subgiants (class IV), and only few giants or supergiants. In Table 1 we give, for each value of M_v, the fraction f of the stars which have types earlier than Sp_d, taken from Trumpler and Weaver (1953).

We define the luminosity function $\phi(M_v)$ for the main sequence alone as $[f\phi_t(M_v)]$, with a small correction for the presence of white dwarfs in ϕ_t. No very accurate data on the frequency of white dwarfs is available, but data on the nearest 50–500 stars indicate that of the order of 10 per cent of all stars are white dwarfs. Luyten (1952) found the luminosity of white dwarfs to be scattered in the range $M_v = +9$ to $+16$. Our adopted luminosity function $\phi(M_v)$ for the main sequence is given in Table 2.

We shall need values for the average mass $\mathfrak{M}$ and absolute bolometric magnitude M_b as a function of M_v for main-sequence stars. The adopted values for the masses, given in Table 2, are a compromise between various compilations (cf. Becker 1950 and Hynek 1951). The adopted mass values are not very accurate but were chosen such that $\mathfrak{M}$ is a smoothly varying function of M_v. The values for M_b, also given in Table 2, were obtained by using Kuiper's (1938*b*) values for the bolometric correction for the average main-sequence spectral type (see Trumpler and Weaver, 1953) corresponding to each M_v.

TABLE 2

LUMINOSITY FUNCTION

M_v	$\log \phi_{tot}+10$	$\log \phi+10$	M_b	$\log(\mathfrak{M}/\mathfrak{M}_\odot)+1$	$\log \psi+10$	$\log \xi+10$
−4	3.58	2.83	−6.64	2.23	5.81	6.63
−3	4.12	3.68	−5.31	2.08	6.28	7.10
−2	4.71	4.41	−3.90	1.93	6.54	7.36
−1	5.32	4.99	−2.31	1.78	6.70	7.52
0	5.99	5.60	−0.90	1.63	6.89	7.72
+1	6.61	6.28	+0.49	1.48	7.16	8.00
+2	6.74	6.55	+1.71	1.33	7.09	7.98
+3	7.02	6.90	+3.00	1.20	7.05	7.98
+4	7.34	7.32	+4.01	1.09	7.32	8.32
+5	7.45	7.45	....	1.00	7.45	8.50
+6	7.50	7.50	....	0.93	7.50	8.60
+7	7.54	7.54	....	0.86	7.54	8.70
+8	7.64	7.64	....	0.80	7.64	8.83
+9	7.76	7.75	....	0.74	7.75	8.97
+10	7.84	7.82	....	0.68	7.82	9.04
+11	7.94	7.91	....	0.62	7.91	9.13
+12	8.02	7.98	....	0.56	7.98	9.20
+13	8.05	8.00	....	0.50	8.00	9.22

III. THE "ORIGINAL MASS FUNCTION"

A portion of the luminosity function of Table 2 is plotted against magnitude in Figure 1. The finer details of this curve are most probably not significant, but the very marked change from a small slope for faint stars to a large slope for bright ones certainly is significant. This change of slope is already significant for ϕ_t, slightly more significant still for ϕ. For instance, $\phi(+8)/\phi(+3)$ is only about 6, as compared with a value of about 150 for $\phi(+3)/\phi(-2)$ for all stars and about 300 for main-sequence stars alone. This change of slope takes place somewhere between $M_v = +1$ and $M_v = +5$ (probably closer to the latter). Also, the main-sequence and giant branches start diverging in about the same magnitude region, which also brackets the magnitude, $M_v \approx +3.5$, at which the main sequence in globular clusters peters out. Further, this change in slope is not due to any peculiar dependence of M_v on variables of greater physical significance like mass. For instance, the mass function, defined as $[\phi dM_v/d(\log \mathfrak{M})]$, plotted against $\mathfrak{M}$, shows a change of slope almost as marked as that of ϕ.

It seems natural to explain the change of slope of the population I luminosity function along lines similar to the explanation for the globular-cluster H-R diagram: There are no stars older than T_0 years (the age of the galaxy), and stars of visual magnitude M_L burn 12 per cent of their hydrogen mass in this time. Hence all stars fainter than this limiting magnitude have burned less and are still on the main sequence. For brighter stars only those sufficiently young to have burned less than 12 per cent of their mass are still on the main sequence, which leads to a rapid decline of main-sequence stars with increasing brightness for $M_v < M_{L,v}$.

More specifically, we wish to test the validity of the series of hypotheses stated at the end of Section I. We defined the "original mass function," $\xi(\mathfrak{M})$, by

$$dN = \xi(\mathfrak{M})\, d(\log_{10}\mathfrak{M})\,\frac{dt}{T_0}, \tag{2}$$

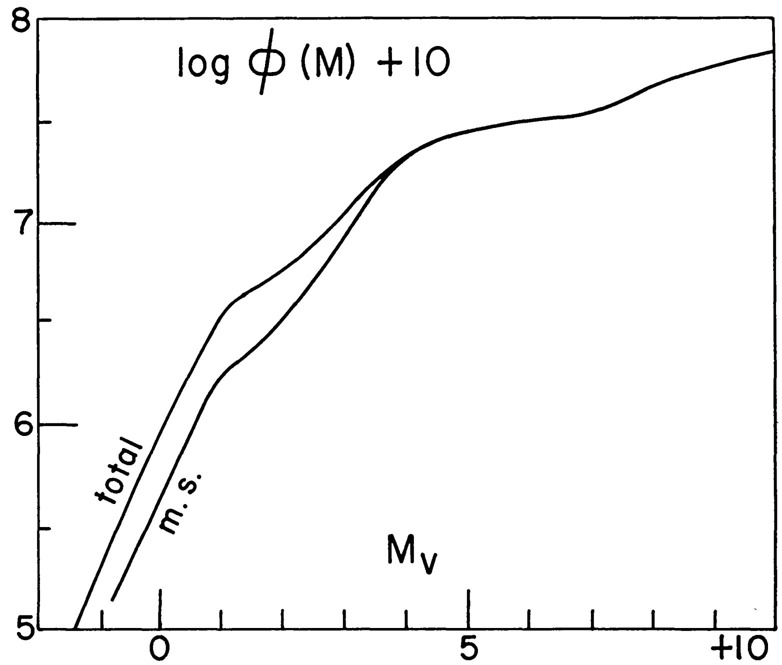

FIG. 1.—The logarithm of the luminosity function ϕ plotted against visual magnitude, M_v. The curve marked *total* denotes the total observed function. The curve marked *m.s.* denotes the derived function for main-sequence stars only.

where dN is the number of stars in the mass range $d\mathfrak{M}$ created in the time interval dt per cubic parsec. The corresponding "original luminosity function," $\psi(M_v)$, is then

$$\psi(M_v) = \xi(\mathfrak{M})\,\frac{d(\log_{10}\mathfrak{M})}{dM_v}. \tag{3}$$

These mass and luminosity functions are, of course, not identical with the observed ones for stars brighter than the limiting magnitude $M_{L,v}$, but are the functions which would be observed if these stars remained on the main sequence forever. Let the average mass, bolometric luminosity, and bolometric magnitude of a main-sequence star of visual magnitude $M_{L,v}$ be $\mathfrak{M}_L$, L_L, and $M_{L,b}$, respectively. The time T spent on the main sequence by a brighter star (time taken to burn 12 per cent of the mass) is then $T_0(\mathfrak{M}/L)$

$(L_L/\mathfrak{M}_L)$. The relation between our "original luminosity function," $\psi(M_v)$, and the observed one, $\phi(M_v)$, is then

$$\log \phi\,(M_v) = \log \psi\,(M_v) + 0.4\,(M_b - M_{L,\,b}) + \log\left(\frac{\mathfrak{M}}{\mathfrak{M}_L}\right) \tag{4}$$

for $M_v < M_{L,\,v}$; $\phi = \psi$ for $M_v > M_{L,\,v}$.

Since the data on $\phi(M_v)$ are not sufficiently accurate to obtain a precise value of $M_{L,\,v}$ from the change of slope, we assume the value indicated by the globular-cluster data of $M_{L,\,v} = 3.5$. Using equation (4) and the values for mass $\mathfrak{M}$ and bolometric magnitude M_b given in Table 2, $\psi(M_v)$ was derived from the observed $\phi(M_v)$. Finally, using equation (3) and the adopted mass values, the function $\xi(\mathfrak{M})$ was derived. Both ψ and ξ are given in Table 2. A plot of ξ against $\mathfrak{M}$ is given in Figure 2, passing through all the points of Table 2, except for three points, marked with circles in the figure.

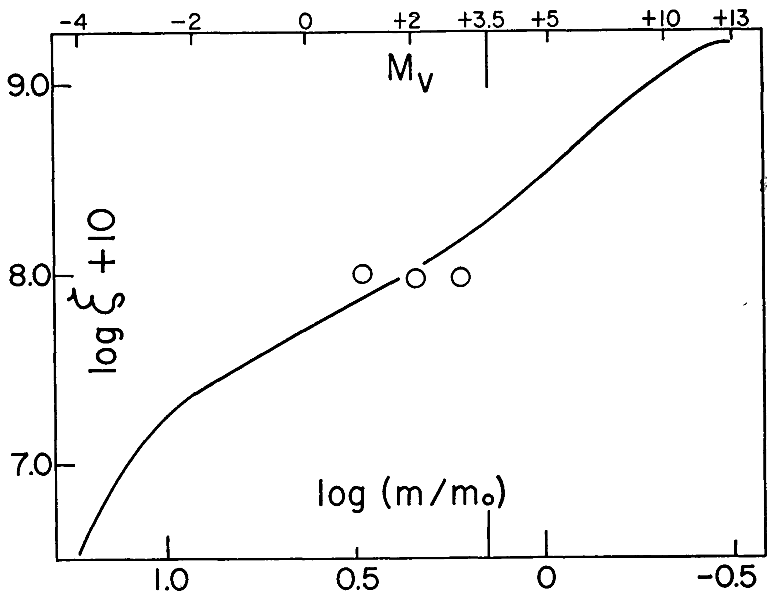

FIG. 2.—The logarithm of the "original mass function," ξ, plotted against the mass, $\mathfrak{M}$, in solar units.

IV. DISCUSSION

Figure 2 and Table 2 show that the "original" mass and luminosity functions ξ and ψ are, in fact, fairly smoothly varying functions without any very rapid change of slope. For log $(\mathfrak{M}/\mathfrak{M}_\odot)$ between -0.4 and $+1.0$, ξ is given reasonably well by the approximation

$$\xi\,(\mathfrak{M}) \approx 0.03\left(\frac{\mathfrak{M}}{\mathfrak{M}_\odot}\right)^{-1.35}. \tag{5}$$

It is not yet clear whether the steeper drop of ξ for masses larger than 10 $\mathfrak{M}_\odot$ is a real effect, since in this region masses and bolometric corrections are not known very accurately and the number of such stars reasonably near the galactic plane is small.

The smoothness of the function ξ lends support to the hypotheses, stated in Section I, on which this paper is based (but of course does not prove them to be correct). Acceptance of these hypotheses, and hence of the mass function $\xi(\mathfrak{M})$, has some interesting consequences. Numerical integration shows that

$$\int_{0.15}^{\infty} d\,(\log \mathfrak{M})\,[\mathfrak{M}\,\xi\,(\mathfrak{M})\,] \approx 0.8 \int_{-0.5}^{0.15} d\,(\log \mathfrak{M})\,[\mathfrak{M}\,\xi\,(\mathfrak{M})\,]\,, \quad (6)$$

$$\int_{0.15}^{\infty} d\,(\log \mathfrak{M})\;\xi\,(\mathfrak{M}) \approx 0.12 \int_{-0.5}^{0.15} d\,(\log \mathfrak{M})\;\xi\,(\mathfrak{M})\,. \quad (7)$$

The left-hand side of equation (6) represents the total mass per cubic parsec of all stars, created at any time since the origin of the galaxy, brighter than limiting magnitude $M_{L,\,v} = 3.5$. Since most of these stars burn their hydrogen very quickly, most of this mass is at present in some form other than main-sequence stars. The integral on the right-hand side of equation (6), on the other hand, is approximately equal to the total mass (per cubic parsec) contained in all stars (brighter than $M_v = 13.5$) which are in existence at present. In other words, the total mass which has been in the form of main-sequence stars once but has taken on different form by now is of the same order of magnitude as the total mass of present stars!

It is interesting to speculate what form this mass of vanished main-sequence stars has taken. One attractive hypothesis is that a massive and luminous star loses most of its mass to the interstellar medium during the last stages of its evolution, the small remnant becoming a white dwarf. Equation (7) lends support to this view: the integrals on the left- and right-hand sides of this equation represent roughly the total number of burned-out and present main-sequence stars, respectively. According to this hypothesis, then, roughly 10 per cent of existing stars should be white dwarfs, which agrees well with observational estimates of the abundance of white dwarfs.

Some workers have estimated that, in regions near the galactic plane, the total mass of existing stars is of the same order of magnitude as the mass of the interstellar gas. If all the assumptions of this paper should turn out to be at least roughly correct, it would then appear likely that an appreciable fraction of the interstellar gas present has at some time passed through the interior of stars. This conclusion, if true, would have some bearing on Hoyle's (1946) theory, which claims that all the chemical elements (except hydrogen) found in the universe today were formed in the interior of stars.

One should perhaps re-emphasize the various sources of error and limitations of the calculations of this paper. The various input data used for deriving the "original mass function" are sufficiently inaccurate so that the values given in Table 2 for log ψ and log ξ may be in error by more than ± 0.20. The numerical coefficients on the right-hand side of equations (6) and (7) could be in error by as much as a factor of 2. Further, the luminosity function discussed in this paper refers only to a relatively small region in the solar neighborhood. It is, of course, unlikely that the same stars and gas clouds have remained in this small region over several billions of years. For this reason, too, the quantitative conclusions of this section should be considered as quite tentative.

Neither the observed nor the "original" luminosity function used in this paper shows a marked sudden drop from the low-luminosity to the high-luminosity side of $M_v \sim 4$. The absence of this drop, so characteristic of population II systems, indicates that the region considered contains mainly population I stars. Nevertheless, a fair fraction of the very faint main-sequence stars and of the white dwarfs could belong to population II.

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